Financial Intermediation and Capital Misallocation

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Abstract

To understand the link between financial intermediation activities and the real economy, we put forward a general equilibrium model where agency frictions in the financial sector affect the efficiency of capital reallocation across firms and generate aggregate economic fluctuations. We develop a recursive policy iteration approach to fully characterize the nonlinear equilibrium dynamics and the off-steady state crisis behavior. In our model, adverse shocks to agency frictions exacerbate capital misallocation and manifest themselves as variations in total factor productivity at the aggregate level. Our model endogenous generate counter-cyclical volatility in aggregate time series and counter-cyclical dispersion of marginal product of capital and asset returns in the cross-section.

Keywords: Financial Intermediation, Capital Misallocation, Volatility, Crisis, Limited enforcement

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I Introduction

Disruptions in financial intermediation activities was a prominent feature of the 2007-2009 great recession and motivated a lot of research to understand the role of financial intermediaries in affecting the real economy.[1] However, whether financial market frictions play a significant role in macroeconomic fluctuations is still a subject of debate. Part of the skepticism is due to the Kocherlakota (2000) critic, which is the observation that financial frictions that affect intertemporal investment is unlikely to generate large economic fluctuations because investment is only a small fraction of the total capital stock. Part of the skepticism comes from the fact that most models with financial frictions feature occasionally binding constraints, but due to the lack of a general global solution method, these models are typically solved by linearizing around a steady-state where the constraints always bind.

The first part of this paper presents a general equilibrium model where agency frictions in the financial intermediary sector affect the real economy not only through the intertemporal investment channel, but more importantly, by affecting the efficiency of capital reallocation among firms with heterogenous productivity. Recent research on capital misallocation, for example, Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), found that large efficiency gains can be achieved by improving capital misallocation, on the order of $30-50\%$. In contrast, intertemporal investment can at most account for $3-5\%$ of total output in standard real business cycle models.[2]

Below we present an additional empirical evidence for the importance of the capital reallocation channel that we focus on in this paper. We show that measured total factor productivity (TFP) is highly correlated with a measure of the efficiency of capital reallocation and the rate of capital utilization.[3] The recent great recession is no exception. In Figure 1, we plot the time series of log TFP (dashed line), measured efficiency of capital reallocation (solid line) and log capital utilization rates (dash-dotted line) in the U.S., where all series are HP filtered. We follow a similar procedure as Hsieh and Klenow (2009) and measure capital misallocation by the variance of the cross-sectional distribution of log marginal product of capital within narrowly defined industries (classified by the four-digit standard industry

[1] Recent surveys and syntheses of this literature include Gertler and Kiyotaki (2010), Quadrini (2011) and Brunnermeier et al. (2012).

[2] In standard RBC models, annual investment is about ten percent of capital stock and capital contributes to roughly one third of total output. According this calculation, the maximum effect of investment on output is about 3.3%.

[3] Capital under-utilization can be interpreted as a special form of misallocation.

classification code) and translate this measure into log TFP units.[4] The measured efficiency of capital reallocation tracks the time series log TFP remarkably closely, indicating that the efficiency of capital reallocation may account for a significant fraction of variations in measured TFP. In addition, economic downturns are typically also associated with sharp declines in in capital utilization rates.

To formalize the link between financial intermediation and capital reallocation, we develop a model of financial intermediation where firms are subject to idiosyncratic productivity shocks and financial contracts cannot be perfectly enforced. The heterogeneity in productivity implies that reallocating capital across firms improves efficiency but requires high productivity firms to borrow from the rest of the economy. Under limited enforcement of financial contracts, capital reallocation must be intermediated in equilibrium because financial intermediaries are better at reinforcing lending contracts than households, who hold the ultimate claim to all assets in the economy. We show that in this setup, declines in financial intermediary net worth increase their incentive to default and limit their borrowing capacity. As a result, shocks that affect financial intermediary net worth impact the efficiency of capital reallocation and manifest themselves in the dispersion of the marginal product of capital and measured aggregate total factor productivity. In the extreme case, when all intermediaries in the economy are constrained, productive capital cannot be fully utilized and output drops sharply.

The second part of the paper develops a recursive method to obtain global solutions of the model. Theoretically, it allows us to construct a Markov equilibrium and characterize the nature of the binding incentive constraints as functions of state variables. Numerically, we develop a recursive policy function iteration approach based on the construction to calibrate our model and evaluate its quantitative implications.

We consider two specifications of out model in calibration: a version with TFP shocks and a version with financial shocks. We show that financial frictions do amplify TFP shocks but the effect is quantitatively small and transitory. Amplification account for about 10% of the variance of aggregate fluctuations in our model with TFP shocks. The lack of magnitude of amplification is due to the inability of productivity shocks to generate substantial movement in asset prices in production economies. Therefore, these shocks do not result in significant variations in bank net worth and the tightness of banks' borrowing constraint. The lack of

[4] We use the formula in Hsieh and Klenow (2009) to translate the variance of log marginal product of capital into a measure of the efficiency of capital reallocation. In Appendix B, we show that this is equivalent to a first order approximation of the efficiency of capital reallocation measured in log TFP units. We detail the data construction in the same appendix.

persistence is because of two offsetting effects. A positive productivity shock not only raises bank net worth but also triggers a surge in investment and financing demand. Because the tightness of banks' borrowing constraint depends on their net worth relative to financing need, the two effect offset each other and the overall impact on the economy is negligible after one or two periods.

Our preferred calibration is the one with financial shocks, which we model as exogenous variations in the discount rate of bank managers. We find that small variations in bankers' discount rate have large and persistent effects on the efficiency of capital reallocation and therefore aggregate output. To understand the mechanism for persistence, note that a positive discount rate shock reduces banker's incentive to make short-term profits and raises bank net worth. The increase in bank net worth improves the efficiency of capital reallocation in the future but does not affect productivity immediately. In response, households consume more and invest less due to the income effect. As a result, increases in bank net worth are associated with temporary reductions in financing needs. These effects reinforce each other, relax banks' limited enforcement constraint, and improve the efficiency of capital reallocation in the next period. More efficient capital reallocation induces a new round of increase in bank net worth and generate long lasting impacts on the economy.

In our benchmark calibration, the standard deviation of banker's discount rate is about 2.3% at the annual level.[5] Nevertheless, the model matches well the macroeconomic moments in the U.S. and produces a volatility of aggregate output of 3.6% from the capital reallocation channel. More importantly, it endogenously generates a counter-cyclical volatility in the time series of aggregate output and consumption, a counter-cyclical dispersion in the cross section of firm output and stock returns, and a counter-cyclical efficiency of capital reallocation and capital utilization as in the data. Our model is also consistent with several salient features of the recent financial crisis, for example, sudden increases in interest spread, sharp drops in capital utilization and capital reallocation, and the slow recovery afterwards.

Our paper belongs to the literature on macroeconomic models with financial frictions.[6] The papers that are most related to our are Gertler and Kiyotaki (2010), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2014), and Rampini and Viswanathan (2014). The

[5]This is much smaller than the variation in discount rates typically found in the asset pricing literature, for example, Campbell and Shiller (1988), and more recently, Lettau and Ludvigson (forthcoming).

[6]A partial list of this literature include Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Kiyotaki and Moore (2005), Bernanke et al. (1999), Krishnamurthy (2003), Kiyotaki and Moore (2008), Mendoza (2010), Gertler and Karadi (2011), Jermann and Quadrini (2012), He and Krishnamurthy (2012), He and Krishnamurthy (2013), and Li (2013). Quadrini (2011) and Brunnermeier et al. (2012) provide comprehensive reviews of this literature.

nature of agency frictions in our model is the same as that in Gertler and Kiyotaki (2010). Different from us, none of the above papers, except Brunnermeier and Sannikov (2014), focuses on the heterogeneity in firms' productivity and the importance of capital reallocation. While the difference in productivity between "households" and "experts" is a key element in the Brunnermeier and Sannikov (2014) model, their analysis is not quantitative, nor do they connect their model with empirical evidences on capital misallocation and under-utilization.

Our approach is related to recent development in using global methods to solve macro models with financial frictions. Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012, 2014), and Maggiori (2013) use continuous time methods to obtain global solutions. Their models all have a single state variable and equilibrium conditions can be reduced to ordinary differential equations, whereas our model involves multiple state variables in order to quantitatively capture a rich set of macroeconomic moments. Mendoza and Smith (2006) study small open economies with margin requirements and use value function iteration to solve their model. We use a policy function iteration approach which greatly improves the numerical efficiency in our general equilibrium setup because it does not involve multiple recursive operators and it uses first order conditions to reduce optimization problems to solving nonlinear equations. Our method can potentially be applied to many other models in this literature, which are often solved using local approximation methods.

Several other papers also emphasize the importance of capital reallocation in understanding credit market frictions. For example, Eisfeldt and Rampini (2006), Eisfeldt and Rampini (2008), Shourideh and Zetlin-Jones (2012), Kurlat (2013), Chen and Song (2013), Fuchs et al. (2013), and Li and Whited (2014). Eisfeldt and Rampini (2006) provide empirical evidence that the amount of capital reallocation is procyclical and the benefit of capital reallocation is counter-cyclical. They also present a model where the cost of capital reallocation is correlated with TFP shocks to rationalize these facts. Eisfeldt and Rampini (2008), Kurlat (2013), Fuchs et al. (2013), and Li and Whited (2014) study adverse selection problems, while we focus on limited enforcement of financial contracts. From the modeling perpective, we differ from the above papers by explicitly allowing for a financial intermediary sector in our model and by using empirical evidence on bank loans and interest rate spreads to discipline our calibration. Quantitatively, we show that relatively small shocks to agency frictions are able to generate quantitatively large macroeconomic fluctuations. Finally, none of the above papers link the countercyclical volatility in aggregate time series to countercyclical dispersion in the cross section in a unified general equilibrium framework.

The idea that shocks may originate directly from the financial sector and affect economic activities is related to the setup of Jermann and Quadrini (2012). Different from Jermann and Quadrini (2012), our paper focus on financial intermediation and capital reallocation and their connections with the macroeconomy.

Our paper is also related to the literature in economics and finance that emphasize the importance of counter-cyclical volatility in understanding the macroeconomy and asset markets. Many authors have documented a strong coutercyclical relationship between real activity and uncertainty as proxied by stock market volatility and/or dispersion in firm level earnings and productivity, for example, Bloom (2009), Bloom et al. (2012), Bachmann et al. (2013), and Jurado et al. (2015), among others. A large literature in asset pricing emphasizes the importance of counter-cyclical volatility in understanding stock market returns, for example Bansal and Yaron (2004), Bansal et al. (2012), and Campbell et al. (2013). Our model generates countercyclical volatility as an endogenous equilibrium outcome even though the primitive shocks are homoscedastic.

The rest of the paper is organized as follows. We provide a summary of some stylized facts that motivate the development of our model in Section II. We describe the model setup in Section III. In Section IV, we discuss the construction of the Markov equilibrium of our model and the recursive policy function iteration approach. In Section V, we analyze a deterministic version of our model to illustrate qualitatively the link between financial intermediation and capital reallocation. We calibrate our model and evaluate its quantitative implications on macroeconomic quantities and asset prices in Section VI. Section VII concludes.

II Stylized Facts

In this section, we provide some stylized facts that motivate the development of our theoretical model. The first fact is about the business cycle properties of the total volume of intermediated loans:

1. The total volume of bank loans is procyclical. It is negatively correlated with measures of volatility and capital misallocation.

   The above fact is what motivates our theory of financial intermediation and its connection with capital reallocation. We calculate the total volume of bank loans of the non-financial corporate sector in the U.S. from the Flow of Funds Table. Total bank loans are calculated as the difference between total corporate credits and corporate bond issuance. The details of the data construction can be found in Appendix B.

We plot the annual changes in the total volume of bank loans and the GDP growth rate of the U.S. economy in Figure 2. The shaded areas indicate NBER defined recessions. It is clear that the total volume of bank loans is strongly procyclical. The correlation between the two series is 0.42 at the annual level.

In Figure 3, we plot the annual changes in the total volume of bank loans and the measured cross-sectional dispersion in the marginal product of capital from the COMPUSTAT data set. We provide the details of the construction of the dispersion measure in Appendix B. Clearly, the innovations of the total volume of bank loans are strongly negatively correlated with our measure of capital misallocation — the correlation of the two series is -0.43 at the annual frequency. This is consistent with the key mechanism of our model: when banks are constrained, the total volume of bank loans decreases, and capital reallocation is less efficient.

We plot the annual changes in the total volume of bank loans and aggregate stock market volatility in Figure 4. Stock market volatility is calculated by aggregating realized variance of monthly returns. The correlation between the two time series is about -0.25 at the annual level. We also plot the cross-sectional dispersion of firm profit in Figure 5. It is clear that changes in the total volume of bank loans is strongly negatively correlated with both measures of volatility.

The rest of the stylized facts are well-known. We therefore do not provide detailed discussion here but refer to the relevant literature. The second fact is about the business cycle properties of capital reallocation. This is documented in Eisfeldt and Rampini (2006).

2. The amount of capital reallocation is procyclical and the cross-sectional dispersion of marginal product of capital is countercyclical.

   The third, fourth and fifth facts are about the cyclical properties of the volatility of macroeconomic quantities and asset returns and are well-known in the macroeconomics literature and the asset pricing literature, for example, Bloom (2009), Bansal et al. (2012) and Campbell et al. (2001).

3. The volatility of macroeconomic quantities, including consumption, investment, and aggregate output is countercyclical.

4. The volatility of aggregate stock market return is also countercyclical. Equity premium and interest rate spreads are countercyclical.

5. The volatility of idiosyncratic returns on the stock market is countercyclical.

In the following sections, we setup and analyze a general equilibrium model with financial intermediation and capital reallocation to provide a theoretical and quantitative framework to interpret the above facts.

III Model Setup

In this section, we describe a general equilibrium model with heterogenous firms and with agency frictions in the financial intermediation sector.

A Non-financial Firms

The specification of non-financial firms in our model follows the standard monopolistic competition setup in the capital misallocation literature, for example, Hsieh and Klenow (2009). There are three types of non-financial firms, intermediate goods producers, final goods producers and capital goods producers. Because non-financial firms do not make intertemporal decisions in our model, we suppress the dependence of prices and quantities on state variables in this subsection.

Final goods are produced by a representative firm on a perfectly competitive market using a continuum of intermediate inputs. We normalize the price of final goods to one and write the profit maximization problem of the final goods producer as:

$$\max \left\{ Y - \int_{[0,1]} p_j y_j dj \right\}$$
$$Y = \left[\int_{[0,1]} y_j^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where p_j and y_j are the price and quantity of input j produced on island j, respectively. Y stands for the total output of final goods. The parameter η is the elasticity of substitution among input varieties. The constant return to scale technology and the fact that the final goods market is perfectly competitive imply that final goods producers earn zero profit in equilibrium. In this case, final goods producer's demand function for input variety j can be written as:

$$p_j = \left[\frac{y_j}{Y} \right]^{-\frac{1}{\eta}}. \tag{2}$$

There is continuum of monopolistically competitive intermediate goods producers indexed by $j \in [0,1]$, each producing a different variety on a separate island.[7][8] We use j as the index for both the intermediate input and the island on which it is produced. The profit maximization problem for the producer on island j is given by:

$$D_F(j) = \max\{p_j y_j - MPK_j \cdot k_j - MPL \cdot l_j\}$$
$$subject\ to: \qquad p_j = [y_j/Y]^{-\frac{1}{\eta}}$$
$$y(j) = \bar{A} a_j k_j^{\alpha} l_j^{1-\alpha}. \tag{3}$$

Here, the production of variety j requires two factors, capital k_j and labor l_j. $\bar{A}$ is the aggregate productivity common across all firms. a_j is island j-specific idiosyncratic productivity shock, which we assume to be i.i.d. over time. MPK_j is the rental price of capital on island j and MPL is the *economy wide* wage rate. Because our focus is on capital reallocation across islands with different idiosyncratic productivity shocks, we allow the rental price of capital to be island specific, but assume frictionless labor market across the whole economy. We use $D_F(j)$ to denote the total profit of firm j, which is paid to households as dividend.

We assume, for simplicity, that there are only two possible realizations of idiosyncratic productivity shocks, a_H and a_L. We denote

$$\Pr ob\,(a = a_H) = \pi; \quad \Pr ob\,(a = a_L) = 1 - \pi. \tag{4}$$

We adopt a convenient normalization,

$$\pi a_H^{1-\eta} + (1-\pi)\, a_L^{1-\eta} = 1. \tag{5}$$

As will become clear later, the above condition implies that the average idiosyncratic productivity is one and total output is given by the standard Cobb-Douglas production function, $AK^{\alpha}N^{1-\alpha}$ in the absence of misallocation.

To allow for endogenous variable capital utilization, we assume that current peroid capital can be used for two purposes: producing output and storage. The capital goods producers

[7] In the rest of the paper, we suppress the state space of varieties, $[0,1]$ to save notation.

[8] We use the terminology "island" to emphasize that capital cannot move freely among producers of different input varieties. The details of capital market frictions is introduced in Section C.

maximize profit by operating the following storage technology:

$$D_K = \max_{K_S} \left\{ G\left(K_S, K\right) - QK_S \right\},$$

where K_S is the total amount of current period capital used in the storage technology, $H\left(K_{S,}K\right)$ is a concave and constant return to scale production technology. We use D_K to denote the profit of capital goods producers, which is paid back to household as dividend and Q to denote the market price of capital.

We assume that capital depreciation at a constant rate δ if used for production. Therefore the law of motion of next period capital is

$$K' = G\left(K_S, K\right) + \left(1-\delta\right) K_U + I, \tag{6}$$

where I is the total amount of new investment in the current period. Without loss of generality, we denote $u = \frac{K_u}{K}$ and

$$G\left(K_S, K\right) = g\left(\frac{K_S}{K}\right) K$$

for some concave function $h\left(\cdot\right)$. Using the resource constraint,

$$K_U + K_S = K,$$

equation (6) can be simplified to:

$$K' = \left[h\left(1-u\right) + \left(1-\delta\right) u\right] K + I.^{9} \tag{7}$$

B Household

There is a representative household with log preferences. As in Gertler and Kiyotaki (2010), market is incomplete and household can only invest in a risk-free deposit account with financial intermediaries. We assume (and later verify) that household's utility maximization

[9] It is more common in the literature to assume total depreciation to depend on u and write $K' = \left(1-\delta\left(u\right)\right) K + I$. Our parameterization implies that utilized capital depreciate at a constant rate, which simplifies our numerical analysis. At the same time, it allows us to capiture the same dynamics as the variable capital utilization literature.

problem can be written in a recursive fashion:

$$V(\mathbf{Z}, W) = \max_{C, B_f} \ln C + \beta E\left[V\left(\mathbf{Z}', W'\right)\right]$$

$$C + B_f = W$$

$$W' = B_f R_f(\mathbf{Z}) + \int D_F(j)\left(\mathbf{Z}'\right) dj + \int D_B(j)\left(\mathbf{Z}'\right) dj + MPL\left(\mathbf{Z}'\right). \tag{8}$$

In the above maximization problem, we assume that there exist a vector of Markov state variables $\mathbf{Z}$, the law of motion of which will be specified later, that completely summarize the history of the economy.[10] Taking the equilibrium interest rate $R_f(\mathbf{Z})$, the dividend payment from intermediate goods producers, $\{D_F(j)(\mathbf{Z})\}_{j\in[0,1]}$ and that from the banks, $\{D_B(j)(\mathbf{Z})\}_{j\in[0,1]}$ as given, the household makes its optimal consumption and saving decisions given its initial amount of disposable wealth, W. Household income includes total savings in the bank account, $B_f R_f(\mathbf{Z})$, total dividends (monopolistic rents) from intermediate goods producers, $\int D_F(j)(\mathbf{Z}')\,dj$, total dividend payment from banks, $\int D_B(j)(\mathbf{Z}')\,dj$, and total labor income, $MPL(\mathbf{Z})$. Here we assume that the household is endowed with one unit of labor in every period, which it supplies inelastically to firms.

In our setup, the representative household owns the ultimate claims of all assets in the economy but must delegate its investment decisions in capital markets to financial intermediaries. The household start the current period with total amount of disposable wealth W, and decides the allocation of W between consumption and investment in the risk-free account with banks. To make the intermediation problem non-trivial and prevent the model from collapsing into a single representative agent setup, as in in Gertler and Kiyotaki (2010), we have assumed incomplete market between the household and the intermediary. That is, the only way for the household to invest intertemporally is through a risk-free account with the intermediary. The household cannot buy or sell aggregate state contingent contract with the intermediary.

[10] In another words, we will focus on Markov equilibria with state space $\mathcal{Z}$, where $\mathcal{Z}$ is the set of all possible realizations of $\mathbf{Z}$. We do not explictly specify the state variable Z here. We construct the Markov equilibrium and the state space $\mathcal{Z}$ in Section IV of the paper.

C Financial Intermediaries

There is one financial intermediary on each island.[11] Financial intermediaries are the only agents in the economy who have access to the capital markets.

We assume that the representative household is divided into bankers and workers, and there is perfect consumption insurance between bankers and workers within the household. Under this assumption, banks evaluate future cash flows using the "stochastic discount factor" implied by the marginal utility of the household: [12]

$$M' = \beta \left(\frac{C(\mathbf{Z}', W')}{C(\mathbf{Z}, W)} \right)^{-1} \tag{9}$$

Consider a bank who enters into a period with initial net worth N. It chooses the total amount of borrowing from the household, B_f, amount of borrowing from peer banks, B_I, and the total amount capital stock for the next period K', subject to the following budget constraint:

$$K' = N + B_f + B_I.^{1314} \tag{10}$$

In our model, the total amount of capital for the next period, K' is determined at the end of the current period before the realization of shocks of the next period. That is, we assume one period time to plan as in standard real business cycle models. However, different from the standard representative firm setup, capital can be reallocated across firms after idiosyncratic productivity shocks are realized, which we turn to next.

The market for capital reallocation opens after the realization of aggregate productivity shock A', and idiosyncratic productivity shocks a'. Let $Q(\mathbf{Z}')$ denote the price of capital on the capital reallocation market in state $\mathbf{Z}'$, and let $Q_j(\mathbf{Z}')$ denote the price of capital on an island with idiosyncratic productivity shock a_j for $j = H, L$, in aggregate state $\mathbf{Z}'$. We use $RA_j(\mathbf{Z}')$ to denote the total amount of capital purchased on the reallocation market by intermediary j in state $\mathbf{Z}'$.[15] The total net worth of intermediary j at the end of the next

[11] Because financial intermediaries on each island face competitive capital markets, one should interpret our model as having a continuum of identical financial intermediaries on each island.

[12] See Gertler and Kiyotaki (2010) for details.

[13] For simplicity, we assume there is no capital adjustement cost. In this case, the price of capital is always one.

[14] With a slight abuse of notation, we use B_f as both the amount of saving of the household and the amount of borrowing of the bank. We do so to save notation, because market clearing requires that the demand and supply of bank loans must equal.

[15] We allow $RA(\mathbf{Z}', a')$ to be negative.

period after the repayment of household loan and interbank borrowing is:

$$N_j' = Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')] - Q(\mathbf{Z}')RA_j(\mathbf{Z}') - R_f(\mathbf{Z})B_f - R_I(\mathbf{Z})B_I. \tag{11}$$

Here we allow $Q(\mathbf{Z}')$, $Q_H(\mathbf{Z}')$ and $Q_L(\mathbf{Z}')$ to be potentially different because limited commitment of financial contracts may prevent the marginal product of capital be equalized to the price of capital on the reallocation market when the constraint is binding. The interpretation of (11) is that at the end of the next period, the total value of capital on island j, including the capital purchased in the current period, K' and the capital obtained on the reallocation market, $RA_j(\mathbf{Z}')$, is $Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')]$. The intermediary also needs to pay back the cost of capital obtained on the reallocation market, $Q(\mathbf{Z}')RA_j(\mathbf{Z}')$, and one-period risk-free loans borrowed from the household and other banks, B_f, and B_I.

Note that capital on the reallocation market can only be purchased by issuing a within period interbank loan. This is because the purchase of capital on the reallocation market happens before production and the receipt of payment from local firms, $Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')]$. Figure 6 illustrates the time of events in period t and period $t+1$. At the end of period t, the household has total disposable income W and the total net worth of the intermediary sector is N. The household wealth is allocated between consumption in the current period, C and a risk-free deposit with the banks, B_f. From the bank's perspective, the total net worth and the total consumer loans, B_f are used to purchase capital at price q. At the end of period t, a typical bank purchased K' amount of capital for period $t+1$ production before the realization of the productivity shocks in $t+1$.

Period $t+1$ is divided into four subperiods. In the first subperiod, the aggregate productivity shock A' and the idiosyncratic productivity shock, a' are realized and the capital reallocation market opens. Banks on the high (idiosyncratic) productivity islands have an incentive to purchase more capital on the reallocation market and banks on the low productivity islands have an incentive to sell. Note that transactions on the capital reallocation market must be done by issuing interbank credit, because at this point production has not begun and banks has not receive payment from firms yet. Production happens in the second subperiod, and firms pay back the cost of capital to local banks at the end of the second subperiod.

In the third subperiod, banks payback their interbank loans and household deposit. Importantly, after banks receive payment from local firms and before they pay back loans to creditors, banks have an opportunity to default. Upon default, bankers can abscond with a fraction of their assets, and set up a new bank to operate on some other island. We assume that the amount of asset bankers can abscond with upon default is:

$$\theta Q_j\left(\mathbf{Z}'\right)\left[K' + RA_j\left(\mathbf{Z}'\right)\right] - \omega\left[Q\left(\mathbf{Z}\right)RA_j\left(\mathbf{Z}'\right) + R_I\left(\mathbf{Z}\right)B_I\right]. \tag{12}$$

The total amount of capital on the island is $[K' + RA_j(\mathbf{Z}')]$, where $RA_j(\mathbf{Z}')$ is purchase on the capital reallocation market under within-period interbank loan. Upon default, bankers take away all of the capital on the island, but they can only sell a fraction θ of them on the market. Therefore, upon default, the total receipt of bankers on island j is $\theta Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')]$. The amount $\theta Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')]$ can be viewed as the deadweight loss associated with bankruptcy. Similar to Gertler and Kiyotaki (2010), we assume that bankers have a better technology to enforce contracts then households. This captured by the parameter $\omega \in [0, \theta]$. The interpretation is that in the event of default, a fraction ω of interbank borrowing can be recovered. The case $\omega = 0$ means banks are no better than households in enforcing contracts, and $\omega = 1$ corresponds to the case of frictionless interbank market. The possibility of default implies that the contracting between borrowing and lending banks must respect the following limited enforcement constraint:

$$N_j' \geq \theta Q_j\left(\mathbf{Z}'\right)\left[K' + RA_j\left(\mathbf{Z}'\right)\right] - \omega\left[Q\left(\mathbf{Z}\right)RA_j\left(\mathbf{Z}'\right) + R_I\left(\mathbf{Z}\right)B_I\right], \quad \forall\ \mathbf{Z}'\ and\ \forall\ j, \tag{13}$$

where N' is given by (11). Inequality (13) is the incentive compatibility constraint for banks. It implies that anticipating the possibility of default, lending banks will make sure that the borrowing bankers do not have the incentive to default on loan in all possible states of the world.

In the last subperiod, bankers clear their interbank transactions and consumers receive dividend payment from banks and firms, risk-free return from bank deposit and make their consumption and saving decisions. At this point, bank net worth is allowed to move freely across islands.

As in Gertler and Kiyotaki (2010), the assumption that bank net worth moves freely at the end of every period is made for tractability. It implies that the expected return on all islands are equalized and therefore the ratio of bank net worth to capital must be equalized across all islands. As a result, the decision problems for banks on all islands are identical at the end of the last subperiod. This allows us to use the optimal decision problem of the representative bank to construct the equilibrium. Without this assumption, bank net worth depends on the history of the realization of idiosyncratic productivity shocks and the distribution of bank net worth across islands become a state variable in the construction of Markov equilibria. In our setup, the heterogeneity in the realization of idiosyncratic productivity shocks at the beginning of a period motivates the need for capital reallocation. At the same time, the possibility of moving bank net worth across islands at the end of a period avoids the need to keep track of the the distribution of bank net worth across islands.

We note that no arbitrage on the capital markets within an island implies that

$$Q_j(\mathbf{Z}') = MPK_j(\mathbf{Z}') + 1 - \delta. \tag{14}$$

The interpretation is that one unit of capital on island j produces an additional current period output $MPK_j(\mathbf{Z}')$ in the current period and depreciates at rate δ after production. In a frictionless market the above condition and the fact $Q_j(\mathbf{Z}') = Q(\mathbf{Z}')$ for all j guarantees that the marginal product of capital must be equalized across all islands. In our model, misallocation may happen in equilibrium due to limited enforcement of financial contracts.

As is standard in the dynamic agency literature, for example, DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007), we assume that bank managers are less patient than households and use Λ to denote the ratio of bankers' discount rate relative to that of the households'. Equivalently, with probability $1-\Lambda$, bankers' net worth is liquidated and paid back to the household as dividend. With probability Λ, where $\Lambda \in (0,1)$ bankers survive to the next period. This assumption is a parsimonious way to capture the idea that the managers of banks have a shorter investment horizon than the representative household and is a necessary condition for agency frictions to persist in the long-run.

Because banks' objective function is linear and the constraints (10), (11), and (13) are homogenous, the value function of banks, taking equilibrium prices as given, must be linear in bank net worth N. In addition, since bank net worth can be freely moved across islands at the end of every period, the marginal value of bank networth must be equalized across all islands at the end of every period. This feature of the model greatly simplifies our analysis, because it implies that banks on different island are just a scaled version of each other after redistribution of bank net worth. We denote the value function of banks as $\mu(\mathbf{Z})N$. A typical bank maximizes:

$$\mu(\mathbf{Z})N = \max_{B_f, K'\{RA_j(\mathbf{Z}')\}_{\mathbf{Z}',j}} E\left[M'\left\{(1-\Lambda(\mathbf{Z}'))N' + \Lambda(\mathbf{Z}')\mu(\mathbf{Z}')N'\right\}\middle|\,\mathbf{Z}\right]$$

by choosing total capital stock for the next period, K', total borrowing from households, B_f, total borrowing from peer banks, B_I, and a state-contingent plan for capital reallocation, $RA_j(\mathbf{Z}')$ for all possible realizations of $\mathbf{Z}'$ and j, subject to constraints (10), (11), and (13).

In our model with financial shocks, we assume that the discount rate, Λ, follows a Markov process. The macro-asset pricing literature found large discount rate variations in the data, one way of interpreting our specification of financial shocks is that we explore the implications of discount rate variations on agency frictions. We show in our calibration that relatively small variations of the discount rate, Λ can be amplified by agency frictions and generate large fluctuations in measured total factor productivity and output.

We make one more assumption on the aggregate productivity $\bar{A}$. We assume $\bar{A}_t = A_t K_t^{1-\alpha}$, where A_t is a Markov process of exogenous productivity shocks. This specification follows Frankel (1962) and Romer (1986) and is a parsimonious way to inject endogenous long-run growth into the model. From a technical point of view, as we will see in Section V of the paper, this allows us to explore homogeneity and reduce one state variable in the construction of the Markov equilibrium.

D Market Clearing

Because market clearing conditions have to hold in every period, we suppress the dependence of all quantities on time and state variables in this section to save notation. We list the resource constraints and market clearing conditions below:

First, the total amount of capital utilized on island j is $K + RA_j, \quad for\ j = H, L$. The resource constraint requires that the amount of capital used fro production on all islands must sum up to uK, which is the total amount of utilized capital in the economy:

$$\pi(K + RA_H) + (1 - \pi)(K + RA_L) = uK. \tag{15}$$

Second, the total amount of interbank borrowing in the economy must be zero. Because banks are ex ante identical before the realization of idiosyncratic productivity shocks, and because interbank borrowing is determined before the realization of these shocks,

$$B_I = 0. \tag{16}$$

The possibility of interbank bank borrowing on the intertemporal bank loan market does not affect allocation but determines the interbank borrowing rate, an object that can be measured empirically and used to discipline our quantitative exercise.

Third, the total net worth of the banking sector equals the sum of bank net worth across all islands:

$$N = \pi N_H + (1 - \pi) N_L. \tag{17}$$

Fourth, labor market clearing requires

$$\pi l_H + (1 - \pi) l_L = 1,$$

because we assumed inelastic labor supply and normalized total labor endowment to one.

Finally, market clearing for final goods requires that total consumption and investment sum up to total output:

$$C + K = Y, \tag{18}$$

where Y is the total output of final goods defined in (1).

Note that market clearing implies that the sum of the household's disposable wealth, W and the total net worth of the banking sector, N must equal to the total financial wealth of the economy. We do not list this condition here because it is redundant given all other market clearing conditions due to Walras' law.

IV Construction of the Markov Equilibrium

A Markov equilibrium consists of i) a set of equilibrium prices and quantities as functions of the state variable $\mathbf{Z}$, and ii) the law of motion of the state variable $\mathbf{Z}$, such that households maximize utility, non-financial firms and financial intermediaries maximize their profit and all markets clear. We follow the following procedure to construct the Markov equilibrium. First, we assume without explicitly specifying the existence of a vector of Markov state variables $\mathbf{Z}$, and derive a set of equilibrium conditions from optimality and market clearing conditions. Second, we explicitly identify the state variables $\mathbf{Z}$ and use equilibrium conditions to contract the law of motion of $\mathbf{Z}$ as well as the equilibrium functionals (equilibrium prices and allocations as functions of $\mathbf{Z}$). Finally, we verify that given the construction of the state variable $\mathbf{Z}$, our proposed pricing functions and quantities constitute a Markov equilibrium. Because our construction of the Markov equilibrium is a recursive procedure, it leads naturally to an iterative procedure to numerically solve the model. We describe our solution method in section C.

We define capital allocation ratio in our model as the ratio of capital employed on high productivity islands relative to that on low productivity islands and denote it as ϕ:

$$\phi = \frac{K + RA_H}{K + RA_L}. \tag{19}$$

It is straightforward to show that the first best level of capital allocation ratio, which we will denote as $\hat{\phi}$ is:

$$\hat{\phi} = \left(\frac{a_H}{a_L}\right)^{\eta-1}.$$

Intuitively, it is optimal to allocate more capital to high productivity islands and less to low productivity islands. The optimal capital allocation ratio is increasing in the elasticity of substitution of output across varieties. The absence of reallocation implies $RA_H = RA_L = 0$ and $\phi = 1$. In general, in our model, $\phi \in \left[1, \hat{\phi}\right]$ and $u \in [0,1]$ summarizes the severity of capital misallocation.

Thanks to the assumption $\bar{A}_t = A_t K_t^{1-\alpha}$, equilibrium quantities are homogenous of degree one in K and equilibrium prices do not depend on K. It is therefore convenient to work with normalized quantities. We define:

$$c = \frac{C}{K},\ i = \frac{I}{K},\ n = \frac{N}{K},\ b_f = \frac{B_f}{K}. \tag{20}$$

Using the above notation, equation (7) can be written as:

$$\frac{K'}{K} = h\left(1-u\right) + \left(1-\delta\right)u + i. \tag{21}$$

Clearly, K must be one of the state variables in the construction of the Markov equilibrium. We denote $\mathbf{Z} = (\mathbf{z}, K)$, where $\mathbf{z}$ is a vector of state variables to be specified later. The homogeneity property implies that normalized equilibrium quantities do not depend on K and only depend on $\mathbf{z}$.

A Equilibrium Conditions

In this section, we analyze the optimality conditions of firms and banks. We provide an aggregation result in Proposition 1, where we show that total output and marginal product of capital can all be represented as functions of capital reallocation ratio, ϕ. Our key result in this section is Proposition 2, which provides a characterization of the nature of binding constraints as a function of state variables.

Product Market Optimality The product market in our model is the standard monopolistic competition setup (see for example, Meltiz 2003). The total output and the marginal product of capital can be represented as functions of u and ϕ, which we summarize in the following proposition.

Proposition 1 *(Aggregation of the Product Market)*

The total output of the economy is given by

$$Y = Auf(\phi)K,$$

where the function $f : \left[1, \hat{\phi}\right] \to [0,1]$ *is defined as:*

$$f(\phi) = \frac{\left(\pi\hat{\phi}^{1-\xi}\phi^{\xi} + 1 - \pi\right)^{\frac{\alpha}{\xi}}}{(\pi\phi + 1 - \pi)^{\alpha}\left(\pi\hat{\phi} + 1 - \pi\right)^{\frac{\alpha}{\xi} - \alpha}} \tag{22}$$

The marginal product of capital on low productivity islands, denoted MPK_L *and the marginal product of capital on high productivity island, denoted* MPK_H *can be written as:*

$$MPK_L(A,\phi) = \alpha\left(1 - \frac{1}{\eta}\right)Af(\phi)\frac{\pi\phi + 1 - \pi}{\pi\hat{\phi}^{1-\xi}\phi^{\xi} + 1 - \pi}, \tag{23}$$

$$MPK_H(A,\phi) = MPK_L(A,\phi)\left(\frac{\hat{\phi}}{\phi}\right)^{1-\xi}, \tag{24}$$

where the parameter $\xi \in (0,1)$ *is defined as* $\xi = \frac{\alpha\eta - \alpha}{\alpha\theta - \alpha + 1}$.

Proof. *See Appendix B.* ■

Note that the function $uf(\phi)$ is a measure of misallocation. It is straightforward to show that f is strictly increasing with $f\left(\hat{\phi}\right) = 1$. In general, $f(\phi) \leq 1$ and misallocation happens when strict inequality holds. Variations in capital misallocation affect $uf(\phi)$ and act like TFP shocks in our model.

The first order condition for capital goods producing firm implies: $Q(\mathbf{z}) = g'(1 - u(\mathbf{z}))$. We use this condition to define Q as a function of u:

$$Q(u) = g'(1 - u). \tag{25}$$

Optimality of Banks' Problem We first simplify the limited enforcement constraints for banks. Combining (12) and (13), the limited enforcement constraint can be written as:

$$(1-\theta)\,Q_H\left(\mathbf{z}'\right)K' - \left[(1-\omega)\,Q\left(\mathbf{z}'\right) - (1-\theta)\,Q_H\left(\mathbf{z}'\right)\right]RA_H\left(\mathbf{z}'\right) \geq R_f\left(\mathbf{z}\right)B_f\left(\mathbf{z}\right), \tag{26}$$

for banks on high productivity islands and

$$(1-\theta)\,Q_L\left(\mathbf{z}'\right)K' - \left[(1-\omega)\,Q\left(\mathbf{z}'\right) - (1-\theta)\,Q_L\left(\mathbf{z}'\right)\right]RA_L\left(\mathbf{z}'\right) \geq R_f\left(\mathbf{z}\right)B_f\left(\mathbf{z}\right), \tag{27}$$

for banks on low productivity islands. We observe that equation (15) and the definition of ϕ and u jointly imply

$$\frac{RA_H}{K} = \frac{u\phi}{\pi\phi+1-\pi} - 1, \quad \frac{RA_L}{K} = \frac{u}{\pi\phi+1-\pi} - 1. \tag{28}$$

Note also that the no arbitrage condition (14) and equations (23,24) imply that $Q_H\left(\mathbf{z}\right)$ and $Q_L\left(\mathbf{z}\right)$ depend on state variables only through (A,ϕ). With a slight abuse of notation, we define

$$\begin{aligned} Q_H\left(A,\phi\right) &= MPK_H\left(A,\phi\right) + 1 - \delta, \\ Q_L\left(A,\phi\right) &= MPK_L\left(A,\phi\right) + 1 - \delta. \end{aligned}$$

If we divide both sides of (26) by K' and use equation (28), we can show that $Q_H\left(A,\phi\right)$ must satisfy:

$$(1-\theta)\,Q_H\left(A',\phi'\right) - \left[(1-\omega)\,Q\left(u'\right) - (1-\theta)\,Q_H\left(A',\phi'\right)\right]\left(\frac{u'\phi'}{\pi\phi'+1-\pi} - 1\right) \geq s', \tag{29}$$

where we denote

$$s' = \frac{R_f b_f}{g\,(1-u) + (1-\delta)\,u + i} \tag{30}$$

Similarly, (27) implies that $Q_L\left(A,\phi\right)$ must satisfy:

$$(1-\theta)\,Q_L\left(A',\phi'\right) - \left[(1-\omega)\,Q\left(u'\right) - (1-\theta)\,Q_L\left(A',\phi'\right)\right]\left(\frac{u'}{\pi\phi'+1-\pi} - 1\right) \geq s'. \tag{31}$$

Let ζ_H and ζ_L denote the Lagrangian multipliers on the limited enforcement constraint, (13). The first order conditions with respect to $RA\left(\mathbf{Z}'\right)$ can be used to derive a relationship between Lagrangian multipliers and the prices of capital on high and low productivity islands.

We use this relationship to define:

$$\zeta_H(A', \phi', u') = \frac{\pi [Q_H(A', \phi') - Q(u')]}{(1-\omega) Q(u') - (1-\theta) Q_H(A', \phi')} \geq 0, \tag{32}$$

$$\zeta_L(A', \phi', u') = \frac{(1-\pi)[Q_L(A', \phi') - Q(u')]}{(1-\omega) Q(u') - (1-\theta) Q_L(A', \phi')} \geq 0. \tag{33}$$

Note that if both of the limited enforcement constraints (29) and (31) hold with equality, then they jointly determine ϕ' and u' as functions of (A', s'). If none of (29) and (31) is binding, then $\zeta_H(A', \phi', u') = \zeta_H(A', \phi', u') = 0$ imply $Q_H(A', \phi') = Q_L(A', \phi') = Q(u')$. Again, ϕ' and u' can be determined as functions of (A', s'). In general, equations (29), (31), (32), (33) and the complementary slackness condition determine ϕ' and u' as functions of (A', s'), which we will denote as $\phi(A', s')$ and $u(A', s')$. The following proposition builds on this observation and characterizes the nature of the binding constraints.

Proposition 2 *(Characterization of Binding Constraints)*
There exists functions $\hat{s}(A)$, $\bar{s}(A)$ and $s^*(A)$, such that

1. *If $s' \leq \hat{s}(A')$, then none of the limited commitment constraints bind, and $\phi(A', s')$ and $u(A', s')$ are determined by (32) and (33).*

2. *If $\hat{s}(A') < s' \leq \bar{s}(A')$, then the limited commitment constraint for banks on high productivity islands binds, and $\phi(A', s')$ and $u(A', s')$ are determined by (30) and (33).*

3. *If $\bar{s}(A') < s' \leq s^*(A')$, then the limited commitment constraint for all banks bind, and $\phi(A', s')$ and $u(A', s')$ are determined by (30) and (31).*

4. *The cutoff levels, $\hat{s}(A')$ and $\bar{s}(A')$ are all increasing functions of A'.*

Proof. *See Appendix C.* ■

The result of the above proposition is intuitive. s' is the total amount of liability that banks need to pay back to households (normalized by capital stock). When s' is below $\hat{s}(A')$, debt level is low enough and the limited enforcement constraints never bind. As debt level increases, when $\hat{s}(A') < s' \leq \bar{s}(A')$, the limited enforcement constraint bind only if the island receives a high productivity shock. Efficiency of capital reallocation requires that banks on high productivity islands borrow more than those on low productivity islands. Therefore, the limited enforcement constraint is more likely to bind for banks on high productivity islands. In the region where $s' > \bar{s}(A')$, the banking sector accumulated too much debt and the limited enforcement constants bind for all realizations of idiosyncratic productivity shocks. Note that the cutoff levels depends on next period aggregate productivity shock, A'. According to the final part of the above proposition, both $\hat{s}(A')$ and $\bar{s}(A')$ are increasing functions of A'; therefore, the limited enforcement constraints are more likely to bind in states where aggregate productivity is low.

The above proposition has two important implications. First, in the cross-section, the limited enforcement constraint is more likely to bind for intermediaries on high productivity islands. This is the mechanism for misallocation in our model: when banks are constrained, more productive projects cannot be financed and measured TFP drops.

Second, in time series, the limited enforcement constraint is more likely to bind when bank net worth is low and/or when aggregate productivity drops. This is the amplification mechanism in our model. Adverse shocks to TFP and bank net worth are amplified because they tighten the limited enforcement constraints and exacerbate capital misallocation.

Given our definition of the Lagrangian multipliers in (32) and (33), we can use other first order conditions to characterize the equilibrium policy functions. Here we use the property that equilibrium prices depend only on $\mathbf{z}$ but not on K to simplify notation. First, the first order condition for households' optimal investment decision leads to the usual intertemporal Euler equation.

$$E\left[M\left(z,z'\right)\right]R_f\left(z\right)=1, \tag{34}$$

where we denote $M(z,z')$ to be the stochastic discount factor of households:

$$M\left(z,z'\right)=\frac{\beta\left[Au\left(z\right)f\left(\phi\left(z\right)\right)-i\left(z\right)\right]}{c\left(z'\right)\left[g\left(1-u\left(z\right)\right)+\left(1-\delta\right)u\left(z\right)+i\left(z\right)\right]}. \tag{35}$$

Also, we introduce a convenient notation, $\widetilde{M}(z,z')$ as

$$\widetilde{M}\left(z,z'\right)=M\left(z,z'\right)\left\{1-\Lambda'+\Lambda'\mu\left(z'\right)\right\} \tag{36}$$

Second, banks' optimal choice for intertemporal investment implies

$$\mu\left(z\right)=E\left[\widetilde{M}\left(z,z'\right)\left\{1+\left(1-\omega\right)\left(\zeta_H\left(A',\phi\left(z'\right),u\left(z'\right)\right)+\zeta_H\left(A',\phi\left(z'\right),u\left(z'\right)\right)\right)\right\}Q\left(u'\right)\right], \tag{37}$$

Third, banks' optimal choice for interbank loan implies

$$\frac{R_I(z)}{R_f(z)} = \frac{E_t\left[\widetilde{M}(z,z')\left\{1+\zeta_H(A',\phi(z'),u(z'))+\zeta_H(A',\phi(z'),u(z'))\right\}\right]}{E_t\left[\widetilde{M}(z,z')\left\{1+(1-\omega)\left(\zeta_H(A',\phi(z'),u(z'))+\zeta_H(A',\phi(z'),u(z'))\right)\right\}\right]}. \tag{38}$$

Fourth, the evenlope condition on banks' optimization problem is

$$\mu(z) = E\left[\widetilde{M}(z,z')\left\{1+\zeta_H(A',\phi(z'),u(z'))+\zeta_H(A',\phi(z'),u(z'))\right\}\right]R_f(z). \tag{39}$$

Finally, we note that the resource constraint requires

$$c(z)+i(z) = Au(z)f(\phi(z)) \tag{40}$$

Note that the four unknown equilibrium functionals, $c(z)$, $i(z)$, $\mu(z)$, and $R_f(z)$ can be determined by the four functional equations (34), (37), (39), and (40). Given the equilibrium functionals, $c(z)$, $i(z)$, $\mu(z)$, and $R_f(z)$, interbank interest rate $R_I(z)$ can be determined by equation (38).

B Construction of the Markov Equilibrium

Subject to some technical details, the four functional equations can be used to determine the four equilibrium functionals, $\{c(z), i(z), \mu(z), R_f(z)\}$ once the law of motion of the state variables are specified. Proposition 2 suggests that it is convenient to include $s' = \frac{R_f b_f}{h(1-u)+(1-\delta)u+i}$ to be one of the state variables. Motivated by this observation, we denote $x = (\Lambda, A)$ to be the vector of exogenous shocks. We conjecture and then verify that a Markov equilibrium can be constructed with $z = (x, s)$ as the state variables. In the rest of this section, we detail the construction of the Markov equilibrium of our model as the fixed point of an appropriate recursive operator.

Because x is an exogenous Markov process, we only need to specify the law of motion of the endogenous state variable, s. Using the law of motion of bank net worth on high and low productivity islands, (11) and the definition of total bank net worth, (17), we can derive the following expression for the law of motion of normalized bank net worth, $n = \frac{N}{K}$:

$$n' = \Lambda'\left\{\alpha\left(1-\frac{1}{\eta}\right)A'u'f(\phi') + (1-\rho')MPK(\rho') + (1-\delta) - s'\right\}. \tag{41}$$

Divide both sides of the bank budget constraint (10) by K to obtain:

$$h(1-u) + (1-\delta)u + i = n + b_f. \tag{42}$$

By the definition of s, we have:

$$s' = \frac{R_f b_f}{h(1-u) + (1-\delta)u + i} = \frac{h(1-u) + (1-\delta)u + i - n}{h(1-u) + (1-\delta)u + i} R_f.$$

Now we can replace n in the above equation using (41) to obtain the law of motion of s:

$$s' = R_f(z)\left\{1 - \frac{\Lambda\left\{\alpha\left(1-\frac{1}{\eta}\right)Au(z)f(\phi(z)) + (1-u(z))MPK(u(z)) + (1-\delta) - s\right\}}{h(1-u(z)) + (1-\delta)u(z) + i(z)}\right\}. \tag{43}$$

Our construction of the Markov equilibrium is formally summarized by the following proposition:

Proposition 3 *(Markov Equilibrium)*

Suppose there exist a set of equilibrium functionals, $\{c(z), i(z), \mu(z), R_f(z)\}_z$ *such that with the law of motion of* s *given by (43),* $\{c(z), i(z), \mu(z), R_f(z)\}_z$ *satisfy the funcitonal equations (34), (37), (39), and (40), then* $\{c(z), i(z), \mu(z), R_f(z)\}_z$ *constitutes a Markov equilibrium.*

Proof. *See Appendix D.* ■

C Recursive Policy Function Iteration

In this section, we describe an operator that maps the space of equilibrium functionals into itself such that if a fixed point for the operator exists, it constitutes a Markov equilibrium described in the last section. There are potentially many such operators. Because the construction of the operator leads naturally to iterative numerical procedures to compute the equilibrium functionals, our construction is aimed toward numerical efficiency.

First, we observe that Proposition 2 allows us to determine the policy functions $\phi(z)$ and $u(z)$ without any iteration. Second, given an initial guess of next period consumption, $c(z)$ and the value of bank net worth, $\mu(z)$, we can use the intertemporal Euler equation (37) to determine the current period consumption and investment policies and use the envelop condition (39) to determine the current period value of bank net worth. At the same time, we need to verify that the policy functions and the law of motion of the state variable, (43) are consistent with each other. Because both (37) and (39) are discounting relationships, it is reasonable to expect that if we iterate this procedure, the policy functions, $c(z)$ and $\mu(z)$ will converge.

Note that our approach makes full use of the first order optimality conditions to improve numerical efficiency. In fact, In Appendix E, we show that the iterative procedure boils down to solving a nonlinear equation for each point in the state space in each step of the iteration. Therefore, numerically, our model involves no more computation than a standard RBC model with productivity shocks. Thanks to the simplification of Proposition 2, the dependence of policy functions on the occasionally binding limited enforcement constraints is fully determined before any iteration. Below is the details of our approach.

1. Using Proposition 2 to construct the policy functions $\phi(z)$ and $u(z)$.

2. Starting from an initial guess of the equilibrium functionals $\{c^0(z), \mu^0(z)\}$.

3. Given a set of equilibrium functionals, $\{c^n(z), \mu^n(z)\}$, let $c(z') = c^n(z')$ and $\mu(z') = \mu^n(z')$ in the definition of $M(z, z')$ and $\widetilde{M}(z, z')$ in equation (35) and (36), respectively. For each z in the state space, solve the four unknows $c(z)$, $\mu(z)$, $i(z)$, $R_f(z)$ from the equations, (34), (37), (39), and (40), with s' defined by (43).

   This is a key step in our iterative procedure. It involves solving four nonlinear equations for four unknowns for each point z in the state space. In the appendix we show that the computation in this step can be reduced to solving a single nonlinear equation for each point z in the state space.

4. Update the equilibrium functionals:

   $$c^{n+1}(z) = c(z), \quad \mu^{n+1}(z) = \mu(z),$$

   where $c(z)$ and $\mu(z)$ are the solutions obtained in step 3.

5. Iterate on step 3 and 4 until the error is smaller than a preset convergence criteria, ε:

   $$\sup_z \left|c^{n+1}(z) - c^n(z)\right| + \sup_z \left|\mu^{n+1}(z) - \mu^n(z)\right| < \varepsilon.$$

Finally, we note that although (x, s) is a convenient choice of state variable that simplifies our construction of the equilibrium and allows for efficient numerical methods to solve the model, any one-to-one function of (x, s) can be used as state variables as well. From an economics point of view, it is more intuitive to use bank net worth as a state variable. Equation (41) defines that mapping between (x, s) and (x, n). We will discuss the implications of our model using (x, n) as the state variable in the rest of the paper.

V Deterministic Dyanmics

In this section, we use the policy functions of the deterministic version of our model to illustrate the mechanism through which bank net worth affects capital misallocation and economic fluctuations. There is no stochastic shocks to x in the deterministic model, and all equilibrium prices and normalized quantities are functions of the normalized net worth n.

A Output, Consumption and Investment

In Figure 7, we plot output (top panel), consumption (middle panel) and investment (bottom panel) as functions of bank net worth n. In the figure, $\hat{n}$ is the level of bank net worth above which the limited enforcement constraints do not bind for any bank and there is no capital misallocation (that is, $\hat{n}$ is the level of net worth corresponds to the $\hat{s}$ defined in Proposition 2). Further increases in bank net worth n do not affect output, consumption and investment because productivity is constant and capital reallocation stay at its first best level. As n decreases towards $\bar{n}$, only the limited commitment constraint for high productivity islands, (26) binds. In this case, as n declines, capital misallocation between high productivity and low productivity islands deteriorates but capital utilization is fully efficient. As n drops below $\bar{n}$, which is the level of net worth corresponds to $\bar{s}$ defined in Proposition 2, the limited enforcement constraint for both islands bind, and output, consumption, and investment drop sharply.

Figure 7 illustrates two key features of our model that continue to hold in the stochastic version. First, total output increases with bank net worth. Note that even in the absence of productivity shocks, when bank net worth is low, the limited commitment constraint (26) and/or (27) binds and limit the efficiency of capital reallocation. As a result, output drops even if factor inputs do not, as if the economy is hit by a negative productivity shock.

Second, the limited commitment constraints are more likely to bind and capital reallocation is less efficient when bank net worth is low. For $n \geq \hat{n}$, output does not depend on bank net worth. In this region of the state space, our model behaves like the frictionless real business cycle model. Productivity shocks (if any) is the only reason for output fluctuations. For $\hat{n} < n \leq \bar{n}$, the limited enforcement constraint on high productivity islands starts to bind and bank net worth affects aggregate output. In the stochastic version of our model, amplification occurs in this region. Negative productivity shocks not only lower output directly through the production function, but also indirectly by reducing bank net worth and the efficiency of capital reallocation. The state space where $n < \bar{n}$ can be intuitively interpreted as the "crisis region". Here the limited enforcement constraints on both types of banks bind. Capital is not only misallocated across high and low productivity firms, but also under-utilized.

B Prices

In Figure 8, we plot the price of capital on the reallocation market (top panel), total wealth of the economy (middle panel), and the interest rate spread (bottom panel) as functions of bank net worth. In the top panel, the market price of capital is determined by the unconstrained firms who equalize their marginal product of capital to its market price. As bank net worth shrinks, the efficiency of capital reallocation deteriorates, and the marginal product of capital of the unconstrained firms drops. At the same time, asset markets are depressed, as we show in the second panel, where we plot the price of an asset that pays aggregate consumption as dividend. In our model bank net worth affect asset prices for two reasons. First, drops in bank net worth affect the efficiency of real production, and as a result, firms cut dividend payment. Second, banks are constrained, and are under pressure to sell. In equilibrium, the market clearing condition implies that asset prices have to decline. In our stochastic model, the two forces reinforce each other to generate large recessions and financial market crisis.

The fact that lower levels of net worth tighten banks' borrowing constraint also manifest itself on the interbank lending market. We plot the spread between interbank interest rate and the household deposit rate in the bottom panel of Figure 8. Interest rate spread widens when the expected return on capital is high, household deposit rate is low, and banks are constrained because of their low net worth. At the same time, banks are less constrained on the interbank market because peer banks have better contract enforcement technologies. As a result, banks race to the interbank lending market and drive up the interest rate, R_I. This effect is particularly pronounced in the crisis region where $n < \bar{n}$, and all banks are constrained.

C Capital Reallocation

As shown in Eisfeldt and Rampini (2006), the amount of capital reallocation is procyclical and the benefit to capital reallocation is countercyclical. Our model is consist with this fact. We plot the dispersion in the marginal product of capital (top panel), the total amount of capital reallocation (second panel), the percentage of capacity utilization (third panel), and the marginal value of bank net worth (bottom panel) as functions of bank net worth in Figure 9. As shown in the top panel of the figure, in the region $n \geq \hat{n}$, capital reallocation is fully efficient, and the marginal product of capital equalize across all islands. As bank net worth decreases to $\bar{n}$, the marginal product of capital on high and low productivity island diverge, but the allocation of capital between low productivity islands and the storage technology is fully efficient. As bank net worth drops further, low productivity islands become constrained as well, and capital "fly to safety", i.e. they are invested in the risk-free storage technology despite its low marginal product. Clearly, the benefit of capital reallocation increases as bank net worth declines.

The divergence of the marginal product of capital is echoed by reductions in the total amount of capital reallocation (second panel) and decreases in the capital utilization rate (third panel). Again, drops in capital reallocation and capital utilization are much more pronounced in the crisis region where the limited enforcement constraints bind for all banks. Finally, we plot the marginal value of bank net worth in the bottom panel of the figure. By the envelope condition (39), the more likely the bank will be constrained in the next period, the higher is the marginal value of bank net worth today. As a result, the marginal value of bank net worth is a decreasing function of n.

D Bank Leverage

We plot the total amount of bank debt (top panel) and bank leverage (middle panel) as a function of bank net worth in Figure 10.

In Figure 11, we plot the next period net worth as a function of current period net worth as the dotted line. The dotted line intersects the 45 degree line, which is the solid line in the figure, only once where its slope is below 45 degree, indicating there is a unique stationary steady state in the model. Bank net worth converge to n_{SSE} in the long run. The solid line is very close to the 45 degree line, especially in the region where both banks are constrained, indicating convergence to steady state is slow and shocks to bank net worth have persistent effects.

In the deterministic model, the economy converges to the steady state n_{SSE} with probability one and stay at the steady state afterwards. In a stochastic model, shocks to TFP and/or discount rates constantly push the system away from the steady state and generate nontrivial economic fluctuations. It is natural to expect the stochastic model to have the following properties. First, negative shocks depress bank net worth and lower the efficiency of capital reallocation. Second, volatility of the economy spikes in the crisis region because output is much more sensitive to shocks that affect bank net worth in this region. Third, capital "flies" to the risk-free storage technology and interest rate spread widens in the crisis region as the limited enforcement constraints bind for all banks. The stock market is depressed not only because expected cash flow drops, but also because intermediaries are constrained and under pressure to sell. We evaluate these effects quantitatively in the next section.

VI Quantitative Results

In this section, we consider two specifications of our model and evaluate quantitatively the impact of financial frictions: a specification with TFP shocks only and a specification with shocks to agency frictions only. In the model with TFP shocks only, productivity shocks are the only source of primitive shocks and are amplified by financial frictions. Consistent with previous literature (for example, Kocherlakota (2000) and Chen and Song (2013)), we find financial frictions do amplify TFP shocks, but the effect is quantitatively small. Amplification account for about 11% of the macroeconomic fluctuations in the model with TFP shocks. In addition, the economy almost never run into the crisis region where the limited enforcement constraint binds for all banks, because TFP shock do not generate large enough variations in bank net worth.

Our preferred calibration is the model with financial shocks, or shocks to agency frictions. In this specification of the model, we introduce stochastic shocks to bankers' discount rate. We show that relatively small shocks generate large fluctuations in capital misallocation and can account for most of the macroeconomic fluctuations in the U.S. economy. We show that this model endogenously generates countercyclical volatility at the aggregate level and countercyclical dispersion in the cross-section. In addition, this version of the model captures several salient features of the recent financial crisis, such as spikes in macroeconomic volatility, sharp drops in capital reallocation and capital utilization, sudden increases in interest rate spreads, "flight-to-quality" or "flight-to-safety", and slow recovery after a financial crisis.

We first choose the model parameters, except the volatility of exogenous shocks to match the first moment of various aspects of the U.S. economy during the period of 1929-2010. We calibrate our model at the quarterly frequency and simulate the model to compute annual moments. We choose the mean productivity level to be $E[\ln A] = 0.1805$ to match a mean growth rate of the U.S. economy of 1.8% per year. We choose $\delta = 2.4\%$ to match a depreciation rate of 10% at the annual level. We choose capital share $\alpha = 0.35$, as is standard in the RBC literature. We calibrate the elasticity of substitution across varieties to be $\eta = 4$, consistent with the estimate in Hsieh and Klenow (2009). We choose the discount rate $\beta = 0.994$ to match a steady-state investment-to-output ratio of 20%. We choose $\frac{a_H}{a_L} = 2.15$ and $\pi = 0.08$ to jointly match the dispersion of TFPR in the U.S. economy reported in Hsieh and Klenow (2009) and the average capital reallcoation rate of 25% reported in Eisfeldt and Rampini (2006).[16]

We choose $E[\Lambda] = 0.97$, consistent with the calibration of Gertler and Kiyotaki (2010). We set $\theta = 0.3026$ to yield a steady leverage ratio of the banking sector of 3.67. consistent with Gertler and Kiyotaki (2010). We set $\omega = 0.0772$, so that the steady interbank interest rate spread in our model matches the historical average of the TED spread (the spread between T-bills and the LIBOR) of 0.64% per year. We choose the capital storage technology to be of the CES form:

$$g(x) = a_0 + \frac{b_0}{\nu} x^{\nu}.$$

We choose a_0 and b_0 to match the time-series average of capital utilization rate of 80% in the data and an average capital depreciation rate of 10% as in RBC models. We choose the elasticity parameter $\nu = 0.98$ so that the volatility of capital utilization rate in our model with financial shocks matches that in the data, 4.08% per year.[17] The calibrated parameters and targeted moments are listed in Table E.

We calibrate $\ln A$ to be an AR(1) process and set the autocorrelation and standard deviation of the AR(1) process to match the volatility and autocorrelation of aggregate output in the data. We set $\Lambda_t = \frac{\exp\{\lambda_t\}}{\exp\{\lambda_t\}+\exp\{-\lambda_t\}}$. This specification allows us to specify λ_t

[16] The simplicity of our model does not allow us to match a rich set of moments TFPR dispersion in the data. Hsieh and Klenow (2009) report that the ratio of the 75th to 25th percentiles of TFPR is 1.7 and that of the 90th to 10th percentiles is 3.3 in the U.S. in 1997. The ratio of the high productivity to low productivity in our model, a_H/a_L is within this range.

[17] The elasticity ν is the only technology parameter that is pin down by a second moment in the data. We choose ν so that the volatility of capital utilization matches our prefered model, which is the one with financial shocks.

as a AR(1) process and grantees that Λ is a valid discount factor for all values of λ_t. We calibrate the autocorrelation and standard deviation of the AR(1) process of λ to match the volatility and autocorrelation of output in the data.

To understand the different implications of TFP shocks and discount rate shocks on financial frictions, we use the policy function iteration method introduced in Section IV to numerically solve the model and plot the impulse functions for shocks to $\ln A$ in Figure 12 and those for shocks to λ in Figure 13, where the solid lines indicate positive shocks and connected dotted lines stand for negative shocks. To emphasize the endogenous persistence generated from our model, we assume all shocks are purely transitory when plotting the impulse response functions. For example, we inject a positive shock into $\ln A$ for one period, and assume that $\ln A$ return to its steady state value immediately after that, even though $\ln A$ is autocorrelated in our calibration.

We make several observations. First, shocks to λ have much more persistent effects than shocks to productivity, even though both shocks occur for one period and return to steady state immediately afterwards. Immediately after a positive productivity shock, bank net worth increases; however, the increase in current period net worth is accompanied by an increase in debt (b_f) of a similar magnitude, this is because high productivity triggers high consumption and high investment at the same time, and as a result, banks must borrow more to finance the additional investment. Because productivity returns back to steady state immediately, so does return to capital. In this case, the initial increase in bank net worth is offset by the increases in interest payment. In fact, as we see in the impulse response functions, banks' own net worth, after service to debt holders drops below steady state. Therefore, TFP shocks do affect bank net worth; however, the effect completely disappears after one period.

The model with λ shocks is completely different in this respect, because increases in bank net worth is accompanied by a change in bank debt in the opposite direction, and the two effects reinforce each other, generating long-lasting impact on the economy. An increase in banker's discount rate reduces dividend payment and increases bank net worth immediately. Because the increase in net worth is not accompanied by increases in productivity, income effect raises consumption immediately and investment drops due to the resource constraint. As a result, bank borrow less from the households. This effect relaxes the limited enforcement constraint going forward, improves capital reallocation, boosts production, and generate a new round of increase in bank net worth. As a result, the initial shock to bank net worth creates a self-reinforcing loop and generates extremely persistent impact. Eventually, it dies off and all quantities converge to steady state. However, the effect is so persistent, that the system is still far from convergence after twenty quarters.

Second, the effect of productivity shocks is largely symmetric: positive and negative shocks in productivity result in changes in quaternities and prices of similar magnitude. Qualitatively, as we have seen in the policy functions in the deterministic case, negative shocks to net worth have larger impact on capital misallocation than positive ones, especially in the "crisis" region. Quantitatively, however, productivity shocks induce very modest changes in bank net worth due to the offsetting effect of bank debt. Although asymmetry and countercyclical volatility is present in this case, they are quantitatively small.

In contrast, the asymmetry in the impulse responses of quantity and prices with respect to shocks to agency frictions is apparent in Figure 13. A positive shock to λ relaxes the limited enforcement constraint and reduces the effect of future shocks. A negative shock to λ tightens the limited enforcement constraint, making the system more sensitive to additional disturbances. As a result, negative shocks are amplified and positive shocks are dampened, leading to endogenous counter-cyclical volatility in our model.

Third, a positive productivity shock is associated with an improvement in capital utilization (i.e., increases in u) but a deterioration in capital reallocation (i.e., drops in ϕ). Upon impact, increases in A attract more capital from the storage technology into the productive sector and raises capital utilization rate, u. However, more capital go into the the low productivity firms because their limited enforcement constraint is not binding. As a result, the efficiency of capital reallocation among productive firms deteriorates even though more capital is deployed in the productive sector. Overall, the efficiency of capital reallocation as measured by $uf(\phi)$ improves but the effect is quantitatively small. Because bank net worth quickly drops back to the steady state level, so does the efficiency in capital reallocation.

A positive innovation in financial shocks, on the other hand, improves the efficiency of capital reallocation and capital utilization at the same time. The two effects reinforce each other, leading to pronounced and persistent changes in total output. At the same time, the Lagrangian multipliers on the limited enforcement constraints shrink and interbank interest spread declines.

To understand the quantitative implications of the model, we simulate the model for 800 quarters and discard the first 400 quarters, aggregate the quarterly quantities in the remaining part of the simulation into annual quantities, and compute moments for annualized quantities. We report moments of macroeconomic quantities in the data and the our models in Table 2. Both specifications of our model are calibrated to match the mean, the volatility, and the autocorrelation of output growth, and the average level of interest rate spread in the data. All other moments are endogenously generated from the model. Both versions of our model are consistent with the basic features of the data in terms of the relatively low volatility of consumption growth, the high volatility of investment, and the comovement between consumption and investment. The level of risk-free interest is too high in both versions of the model — this is the risk-free rate puzzle in production economies, which we do not attempt to address in this paper.[18] Consistent with the data, both versions of our model produces fairly mild volatility of the interbank interest rate spread.

Our model with financial shocks produces a strong countercyclical volatility in aggregate time series, while the model with TFP shocks does not. In Table 2, the notation $Corr[\Delta lnY, Vol(\Delta lnY)]$ stands for the correlation between current period output growth and the realized variance of future output growth. For each year, we compute the realized variance of future output growth in the data as the realized variance of the growth rates of quarterly industrial production during the next two years. In the model, we compute it as the realized variance of output growth for the next eight quarters in our simulation. As in the data, the correlation between output growth and realized variance of future output growth is stongly negative in our model with financial shocks. However, the same correlation is negligible in the model with TFP shocks. This phenomenon is also evident in the impulse functions we plot for shocks to $\ln A$ (Figure 12) and shocks to λ(Figure 13). As we explain before, symmetric shocks in $\ln A$ produces roughly symmetric responses in total output, consumption and investment, as in standard neoclassical models, while negative shocks to λ produces a significantly larger effect of total output than positive shocks.

Note that measured log TFP in our model equals $\ln \bar{A}_t + \ln u_t f(\phi_t)$, where the component $\ln u_t f(\phi_t)$ depends on the efficiency of capital reallocation. In the last row of Table 2, we report the fraction of the realized variance of TFP growth that comes from variations in capital misallocation in out models:

$$\frac{Var\left[\ln\left(u_{t+1} f\left(\phi_{t+1}\right)\right) - \ln\left(u_t f\left(\phi_t\right)\right)\right]}{Var\left[\ln \bar{A}_{t+1} - \ln \bar{A}_t + \ln\left(u_{t+1} f\left(\phi_{t+1}\right)\right) - \ln\left(u_t f\left(\phi_t\right)\right)\right]}.$$

In the model with TFP shock, the efficiency of capital reallocation account for 11% of total variation in TFP. Therefore, amplification is present in this version of the model, but is quantitatively small. In the model with financial shocks only, the efficiency of capital

[18] Ai et al. (2013) show that this issue can be resolved by using a recursive utility with high intertemporal elasticity of substitution.

reallocation account for virtually all of the macroeconomic fluctuations.

We document the statistics related to the quantity and benefit of capital reallocation in Table 3. Both versions of our model are consistent with the empirical evidence of procyclical capital reallocation and procyclical capital utilization. However, consistent with small magnitude of amplification, the variations in capital reallocation and capital utilization in the model with TFP shocks is much smaller compared to the data and compared to our model with financial shocks. In addition, the cross-sectional dispersion of the marginal product of capital is positively correlated with measured TFP, while this correlation is negative both in our model and in the data, consistent with the empirical evidence in Eisfeldt and Rampini (2006). The reason that TFP shocks generate procyclical benefit of capital reallocation is that positive TFP shocks move more capital from the risk-free storage technology to the productive sector, but most of the capital goes to the less productive firms whose limited enforcement constraint does not bind. As a result, although positive TFP shocks improve capital utilization, they also elevate the cross-sectional dispersion of the marginal product of capital, as shown in Figure 12.

To further understand the implications of our model on volatility dynamics and economic recessions, we report the moments of macroeconomic quantities and interest rate spreads in the data and those in our model for recession periods and for non-recession periods separately. For simplicity, we use a "rule of thumb" classification and define recession as two consecutive quarters of declines in real GDP both in the data and in the model. Our definition yields very similar results as the NBER definition of recession, and results in about 20% of the sample being classified as recession both in the data and the model simulation.

Clearly, as in the data, the volatility of consumption and output are strongly countercyclical in our model. Interestingly, there is no significant difference between the volatility of investment in recession periods and that in non-recession periods both in the data and in our model. Spikes in the volatility of output do not lead to significant increases in the volatility of investment in our model, because shocks to λ are stationary. In recessions, bank net worth is low and expected return is high. Negative shocks to bank net worth, although reduces total output, but also raises expected return. The two effect offset each other and do not lead to significant increases in the volatility of investment. Overall, our model with financial shocks is consistent with the pattern of capital utilization and interest rate spread in the data. In recessions, the level of capital utilization drops, but the volatility of capital utilization rates rises. The spread between interbank lending rate and household deposit rate widens, and so does the volatility of the spread. All the above features are the endogenous outcomes of the financial frictions in the model.

VII Conclusion

We presented a general equilibrium model with financial intermediary and capital reallocation. Our model emphasize the role of financial intermediary in reallocating capital across firms with heterogenous productivity. We show that shocks to financial frictions alone may account for a large fraction in the fluctuations of measured TFP and aggregate output. Our calibrated model is consistent with the salient features of business cycle variations in macroeconomic quantities and asset prices. In particular, our model successfully generates countercyclical volatility in aggregate consumption and output, and countercyclical dispersion in the cross-section.

An important next step is to infer or impute shocks to financial frictions from the data and investigate whether our model can account for the realized variations in macroeconomic quantities and asset prices once these shocks are fed into the model. One possible way is to infer financial frictions from the dispersion in the marginal product of capital in the data. The close link between the dispersion measure and TFP in Figure 1 suggests that our model hold promises. A stronger discipline may be imposed on the model if we can infer shocks to θ directly from banks' balance sheet variables. We leave these for future research.

VIII Appendix

A Misallocation and Aggregation on the Product Market

Aggregation

We first derive an aggregation result that is similar to Hsieh and Klenow (2009) and Hopenhayn and Neumeyer (2008). In fact, the product market of our model is a special case of Hsieh and Klenow (2009) and Hopenhayn and Neumeyer (2008) without labor market distortions.

Consider the maximization problem in (3), first order conditions with respect to $k(j)$ and $l(j)$ imply:

$$(1-\alpha)\left(1-\frac{1}{\eta}\right)p_j y_j = MPL \cdot l_j \tag{44}$$

$$\alpha\left(1-\frac{1}{\eta}\right)p_j y_j = MPK_j \cdot k_j \tag{45}$$

Together, the above imply:

$$\frac{k_j}{l_j} = \frac{MPL}{MPK_j}\frac{\alpha}{1-\alpha} \tag{46}$$

To save notation, we denote $A_j = Aa(j)$ in this section. Note also, total output of firm j can be written as:

$$y_j = A_j k_j^{\alpha} l_j^{1-\alpha} = A_j \left[\frac{k_j}{l_j}\right]^{\alpha} l_j \tag{47}$$

$$= A_j \left[\frac{l_j}{k_j}\right]^{1-\alpha} k_j. \tag{48}$$

Using (46) and (47), we can write l_j as a function of y_j:

$$l_j = \frac{y_j}{A_j}\left[\frac{\alpha MPL}{(1-\alpha)MPK_j}\right]^{-\alpha}. \tag{49}$$

Similarly, (46) and (48) together implies

$$k_j = \frac{y_j}{A_j}\left[\frac{\alpha MPL}{(1-\alpha)MPK_j}\right]^{1-\alpha}. \tag{50}$$

Using the demand function $p_j = \left[\frac{y_j}{Y}\right]^{-\frac{1}{\eta}}$, we can replace y_j in the above equations by $p_j^{-\eta}Y$, and integrate across all j, we have:

$$\bar{K} = \int \frac{p_j^{-\eta}}{A_j} \left[\frac{1}{MPK_j}\right]^{1-\alpha} dj \left[\frac{\alpha MPL}{1-\alpha}\right]^{1-\alpha} Y \tag{51}$$

$$\bar{L} = \int \frac{p_j^{-\eta}}{A_j} \left[\frac{1}{MPK_j}\right]^{-\alpha} dj \left[\frac{\alpha MPL}{1-\alpha}\right]^{-\alpha} Y, \tag{52}$$

where $\bar{K}$ and $\bar{L}$ stands for the total capital and total labor employed for production, respectively. Together, (51) and (52) imply

$$Y = \frac{\bar{K}^{\alpha}\bar{L}^{1-\alpha}}{\left[\int \frac{p_j^{-\eta}}{A_j}\left[\frac{1}{MPK_j}\right]^{1-\alpha} dj\right]^{\alpha}\left[\int \frac{p_j^{-\eta}}{A_j}\left[\frac{1}{MPK_j}\right]^{-\alpha} dj\right]^{1-\alpha}}. \tag{53}$$

We can express p_j in (53) by functions of productivity and prices. Note that (44) and (45) imply

$$MPK_j \cdot k_j + MPL \cdot l_j = \left(1 - \frac{1}{\eta}\right) p_j y_j. \tag{54}$$

Using (49) and (50), we have:

$$MPK_j \cdot k_j + MPL \cdot l_j = \frac{y_j}{A_j}\left[\frac{MPL}{(1-\alpha)}\right]^{1-\alpha}\left[\frac{MPK_j}{\alpha}\right]^{\alpha}. \tag{55}$$

Combining (54) and (55), we have:

$$p_j = \frac{\eta}{\eta-1}\frac{1}{A_j}\left[\frac{MPL}{(1-\alpha)}\right]^{1-\alpha}\left[\frac{MPK_j}{\alpha}\right]^{\alpha}. \tag{56}$$

Note that the normalization of price we choose in (2) implies $\int p_j dj = 1$. Integrating (56) over j, we have:

$$\frac{\eta}{\eta-1}\left[\frac{MPL}{(1-\alpha)}\right]^{1-\alpha} = \left\{\int \frac{1}{A_j}\left[\frac{MPK_j}{\alpha}\right]^{\alpha} dj\right\}^{-1}. \tag{57}$$

Together, (56) and (57) imply

$$p_j = \frac{\frac{1}{A_j}\left[\frac{MPK_j}{\alpha}\right]^{\alpha}}{\int \frac{1}{A_j}\left[\frac{MPK_j}{\alpha}\right]^{\alpha} dj}. \tag{58}$$

Replacing p_j in equation (53) with (58), and using $A_j = A^{1-\alpha}a(j)$, we can write $Y =$

$TFP\bar{K}^{\alpha}\bar{L}^{1-\alpha}$, where

$$TFP = A\frac{\left\{\int\left(\frac{a_j}{MPK_j^{\alpha}}\right)^{\eta-1} di\right\}^{\frac{\eta}{\eta-1}+\alpha-1}}{\left\{\int\left(\frac{a_j}{MPK_j^{\alpha}}\right)^{\eta-1} \frac{1}{MPK_j} di\right\}^{\alpha}}. \tag{59}$$

Under the assumption (**??**), it is straightforward to show that $TFP = A$ is $MPK_j = MPK$ for all j. We define

$$EF = \frac{\left\{\int\left(\frac{a_j}{MPK_j^{\alpha}}\right)^{\eta-1} di\right\}^{\frac{\eta}{\eta-1}+\alpha-1}}{\left\{\int\left(\frac{a_j}{MPK_j^{\alpha}}\right)^{\eta-1} \frac{1}{MPK_j} di\right\}^{\alpha}} \tag{60}$$

to be the efficiency measure of capital reallocation. Under the assumption $\ln \alpha_j$ and $\ln MPK_j$ are jointly normally distributed, we can show that

$$\ln EF = -\frac{1}{2}\left[\alpha\left(\eta-1\right)+1\right]\alpha\sigma^2, \tag{61}$$

where σ^2 is the cross-sectional variance of marginal product of capital. Note also, (61) is approximately true for arbitrary distributions as long as the deviation of $\ln \alpha_j$ and $\ln MPK_j$ from there mean is small. Therefore, (61) can be viewed as a first order Taylor approximation that maps the cross-sectional variance of marginal product of capital into TFP losses due to misallocation.

Proof of Proposition 1

In the special case where a_j takes on only two values, a_H and a_L as in (4), we define $\phi = \frac{K_H}{K_L}$ to be the ratio of capital employed on islands with high productivity shock with respect to that employed on islands with low productivity shock, as in (**??**). Note that

$$MPK_j = \alpha A a_j \left(\frac{l_j}{k_j}\right)^{1-\alpha}; \quad MPL = \left(1-a\right) A a_j \left(\frac{k_j}{l_j}\right)^{\alpha}.$$

Note that because labor market is perfectly mobile, MPL must equalize across all islands. Using the labor market clearing condition, (**??**) and assumption (5), we can prove conditions (23) and (24). Using there conditions to replace MPK_j in (60), the efficiency measure (60) can be written as (22). This completes the proof of Proposition 1.

B Data Construction

B.1 Misallocation and TFP

In Figure 1, we plot the measure of capital misallocation and total factor productivity. We measure the cross-sectional dispersion of TFPR following Hsieh and Klenow (2009). In the context of our model, equation (45) implies

$$MPK_j = \alpha\left(1 - \frac{1}{\eta}\right)\frac{p_j y_j}{k_j}.$$

Following Chen and Song (2013), we measure MPK_j by the ratio of Operating Income before Depreciation (OIBDP) to one-year-lag net Plant, Property and Equipment (PPENT). As in Hsieh and Klenow (2009), we focus on the manufacturing sector and compute the cross-sectional dispersion measure within narrowly defined industries (as classified by the 4-digit standard industry classification code). Specifically, for firm j in industry i, we compute

$$\frac{MPK_{i,j}}{MPK_i} = \frac{\alpha\left(1-\frac{1}{\eta}\right)\frac{p_{i,j}y_{i,j}}{k_{i,j}}}{\alpha\left(1-\frac{1}{\eta}\right)\frac{p_j y_j}{k_j}} = \frac{\frac{p_{i,j}y_{i,j}}{k_{i,j}}}{\frac{p_j y_j}{k_j}},$$

where $\frac{p_j y_j}{k_j}$ is measured at the industry level. We than compute the variance of $\frac{MPK_{i,j}}{MPK_i}$ for each year. This is our empirical measure of σ^2 in equation (61). We use the first order approximation in (61) to construct the time series of the misallocation measure, which is the solid line in Figure 1. The measure of total factor productivity is directly taken from the published TFP series on the Federal Reserve Bank of St Louis website. Both series are HP filtered.

B.2 Total Volume of Bank Loans

We measure the total volume of bank loans of non-financial corporate sector through the aggregate balance sheet of nonfinancial corporate business (Table B.102) as reported in the U.S. Flow of Funds Table. In particular, the bank loan is calculated as the difference between total credit market liability (Line 23) and corporate bond (Line 26). Under this construction, bank loans consist of the following credit market liability items: commercial paper (Line 24), municipal securities (Line 25), depository institution loans (Line 27), other loans and advances (Line 28) and mortages (Line 29).

C Characterization of Binding Constraints

D Constuction of the Markov Equiilbrium

E Computation Details

Figure 1: **Log TFP and Capital Misallocation Measured in Log TFP Units**

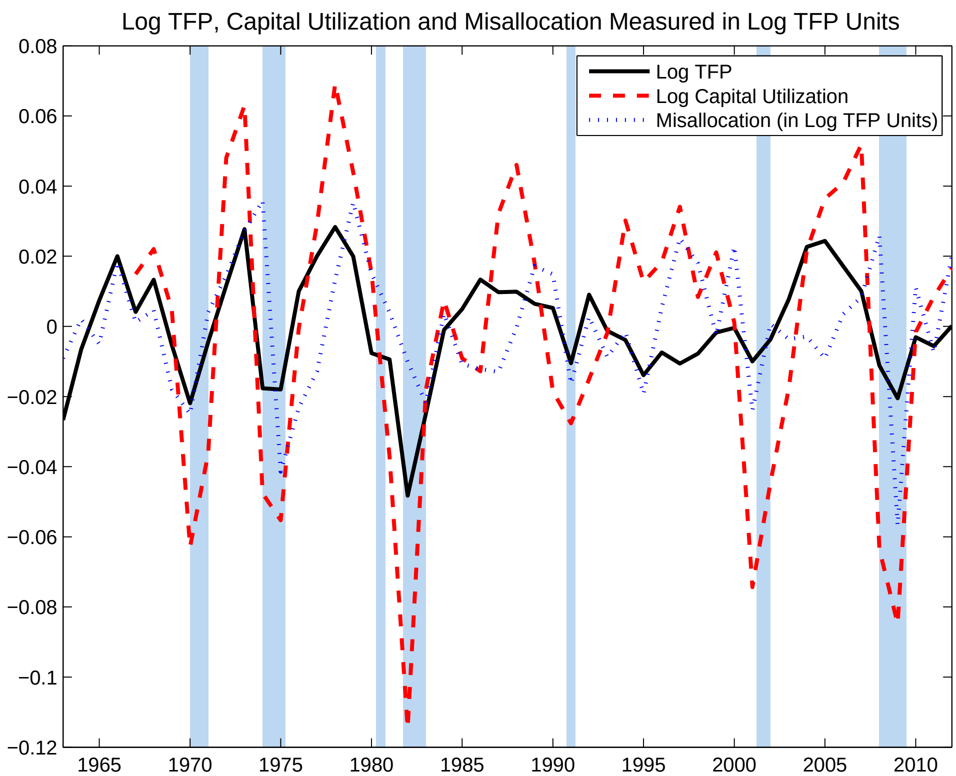

Figure 1 plots the time series of total factor productivity (dashed line) in the U.S. and the measure of capital misallocation (solid line) in the period 1963-2012. The construction of the misallocation measure follows Hsieh and Klenow (2009). We provide the details of the construction in Appendix B. We use the first order Taylor expansion in equation (61) to translate the misallocation measure into log TFP units. Both series are HP filtered.

Figure 2: **Business Cycle Variations of the Total Volume of Bank Loan**

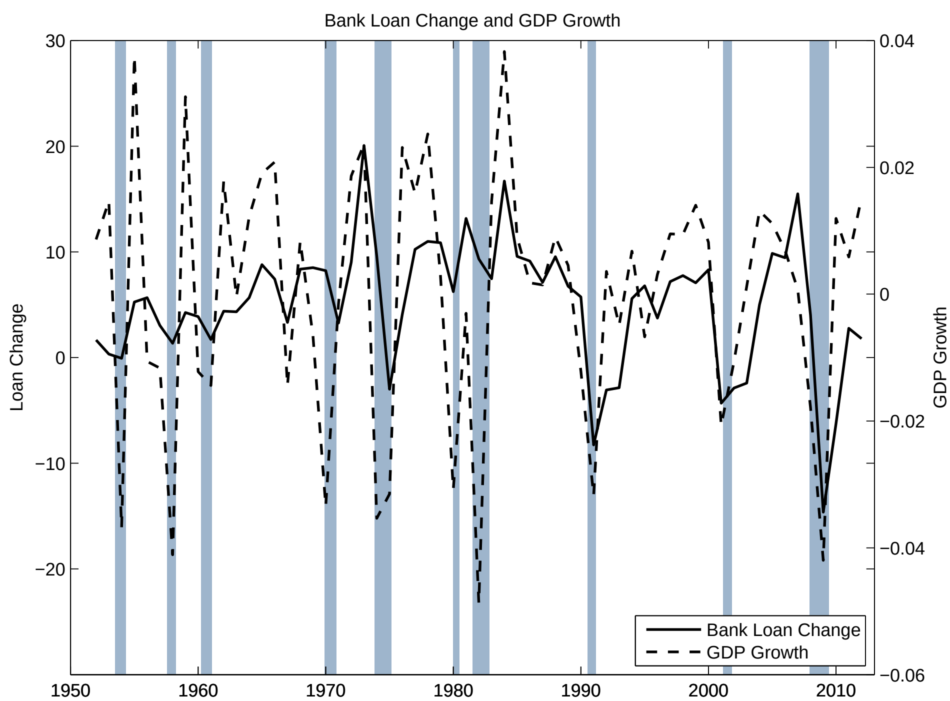

Figure 2 plots the business cycle variations of the total volume of bank loans for all non-financial firms in the US corporate sector. The solid line is the changes in the total volume of bank loans and the dashed line is GDP growth. Shaded areas stand for NBER classified recessions.

Figure 3: **Total Volume of Bank Loan and Capital Misallocation**

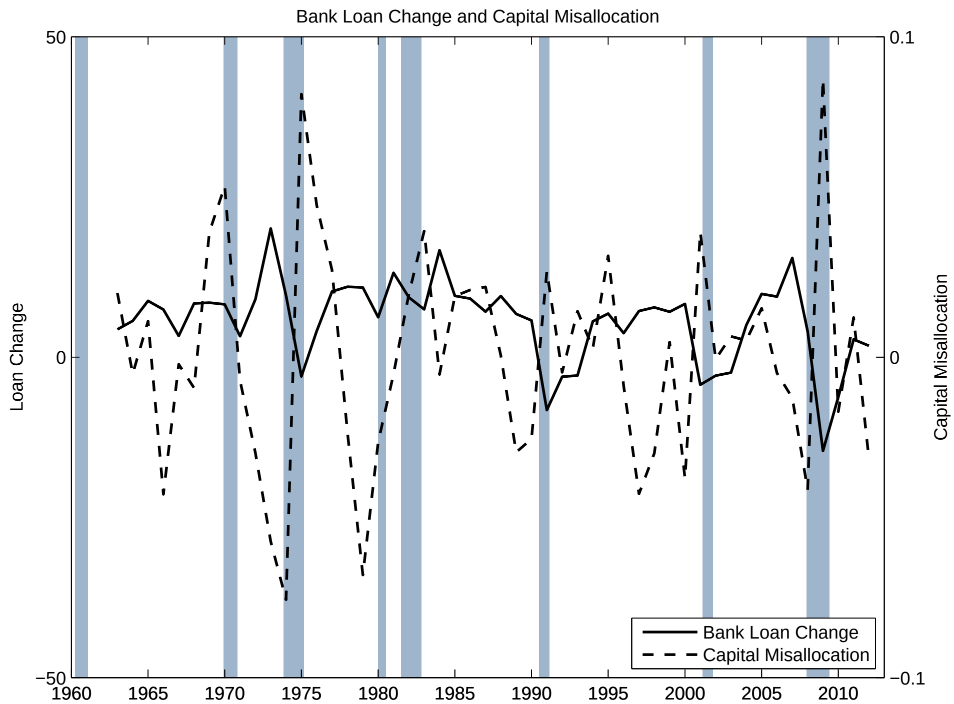

Figure 3 plots the net increases in the total volume of bank loan and our measure of capital misallocation constructed from COMPUSTAT firms during the period 1958-2012.

Figure 4: **Total Volume of Bank Loan and Aggregate Volatility**

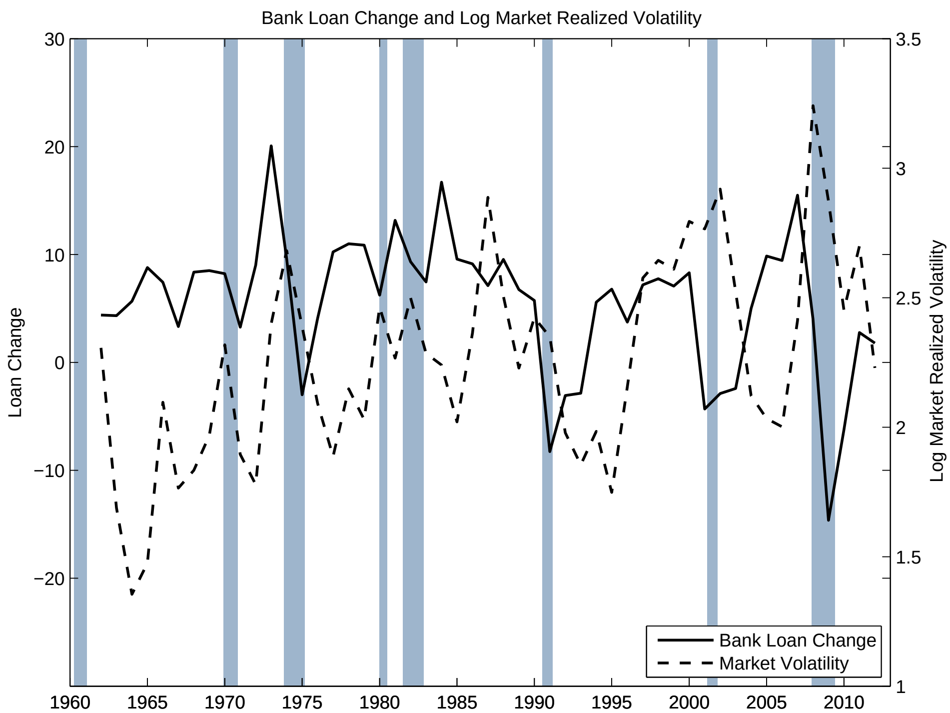

Figure 4 plots the net increases in the total volume of bank loan and stock market volatility in the U.S. during the period 1958-2012.

Figure 5: **Total Volume of Bank Loan and Idiosyncratic Volatility**

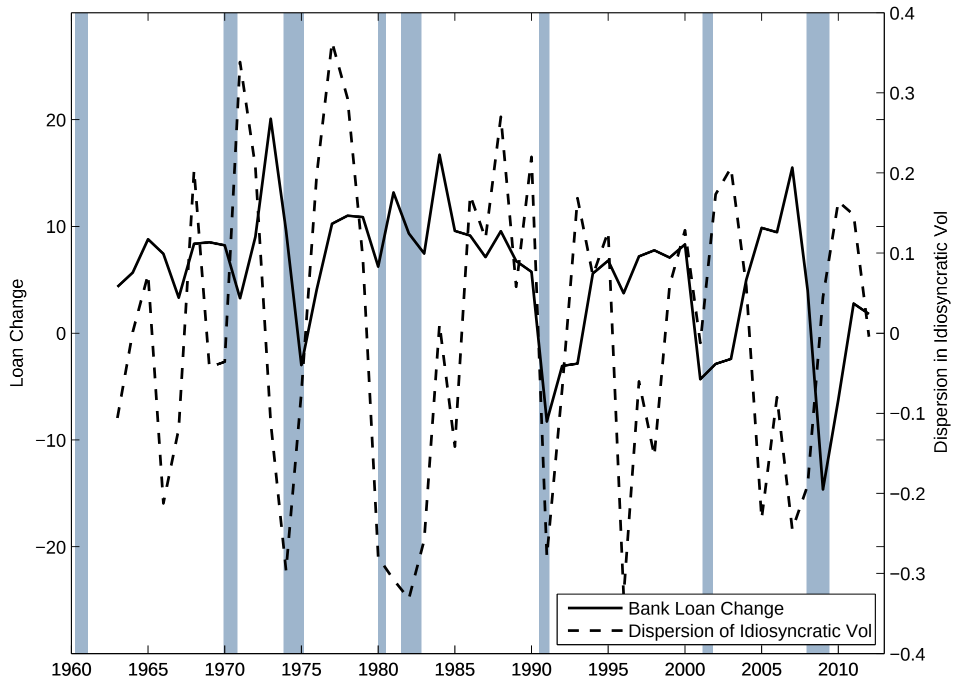

Figure 5 plots the net increases in the total volume of bank loan and the cross-sectional dispersion of firm profit for COMPUSTAT firms.

Figure 6: **Timing of Events**

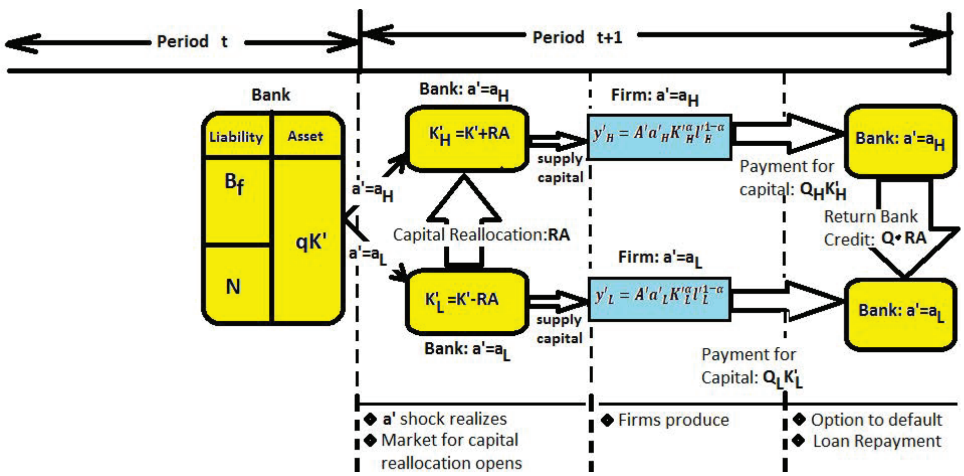

Figure 6 illustrates the timing of event from period t to period $t+1$ in the infinite horizon model.

Figure 7: **Macroeconomic Quantities and Bank Net Worth**

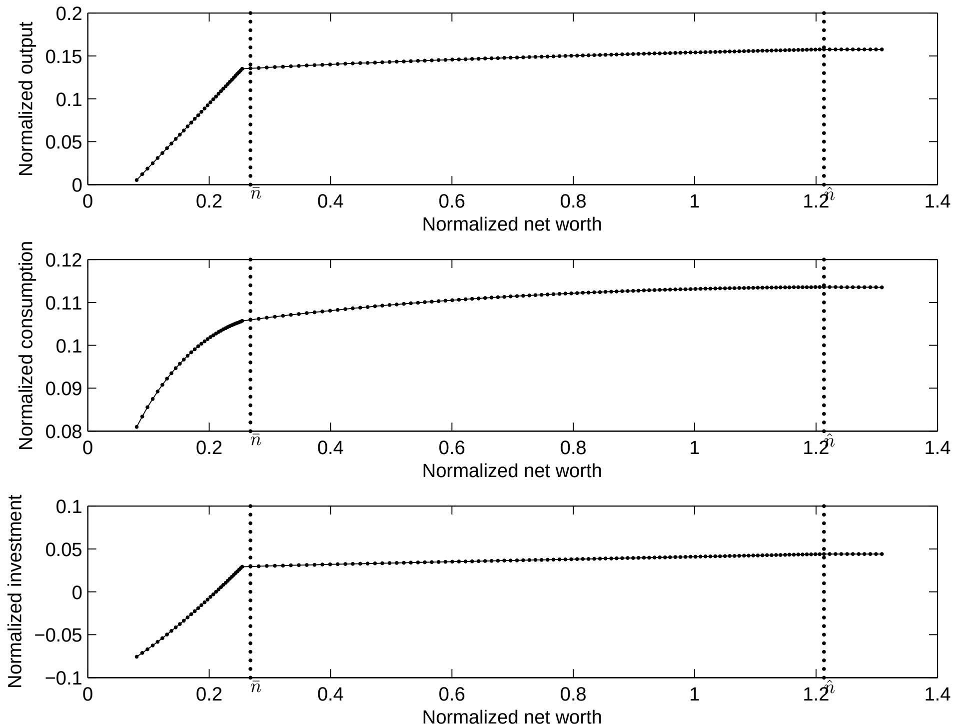

Figure 7 plots the normalized output (top panel), normalized consumption (middle panel) and the normalized investment (bottom panel) as functions of bank net worth. $\hat{n}$ is the cutoff value of bank net worth below which the limited enforcement constraints start to bind for some banks, and $\bar{n}$ is the cutoff value below which the limited commitment constraint bind for all banks.

Figure 8: **Asset Prices and Bank Net Worth**

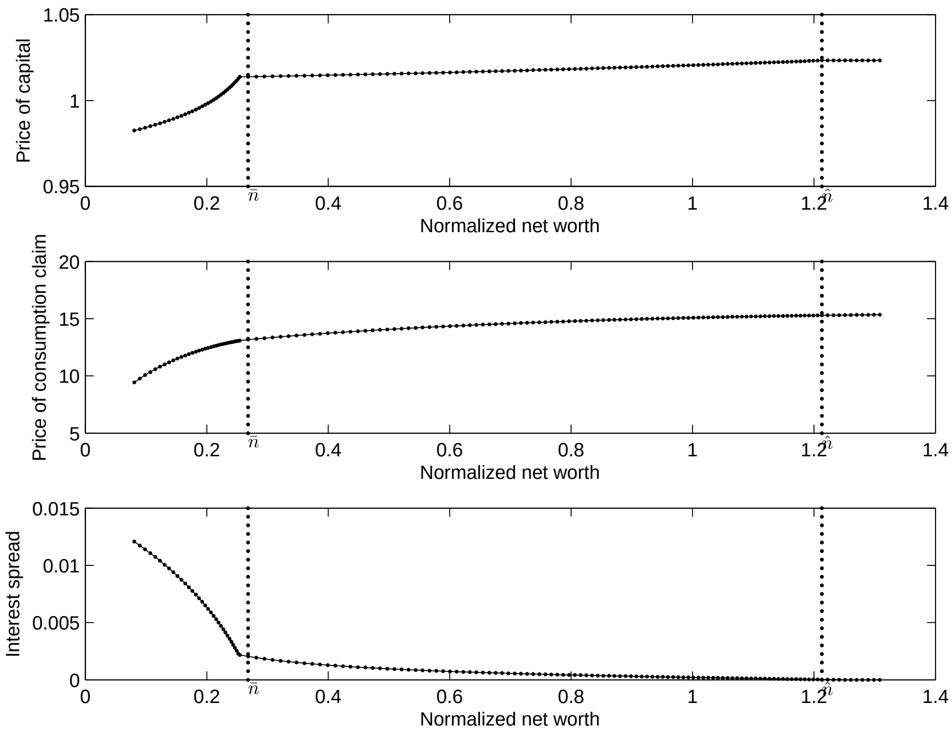

Figure 8 plots the price of capital on the reallocation market (top panel), the price of consumption claim (middle panel), and the spread between interbank lending rate and the household deposit rate (bottom panel) as functions of bank net worth. $\hat{n}$ is the cutoff value of bank net worth below which the limited enforcement constraints start to bind for some banks, and $\bar{n}$ is the cutoff value below which the limited commitment constraint bind for all banks.

Figure 9: **Capital Reallocation and Bank Net Worth**

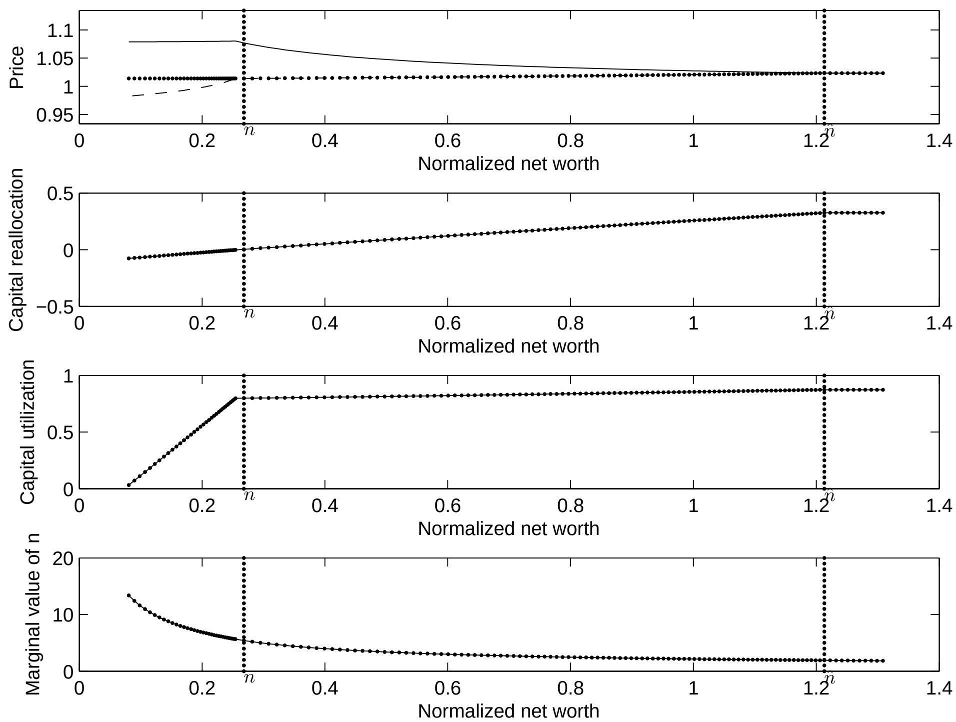

Figure 9 plots the marginal product of capital (top panel), total amount of capital reallocation (second panel), rate of capital utilization (third panel), and the marginal value of bank net worth (bottom panel) as functions of bank net worth. In the top panel, the solid line is marginal product of capital on high productivity islands, the dotted line is that on low productivity islands, and the dashed line is the marginal product of capital in the risk-free storage technology.

Figure 10: **Leverage and Bank Net Worth**

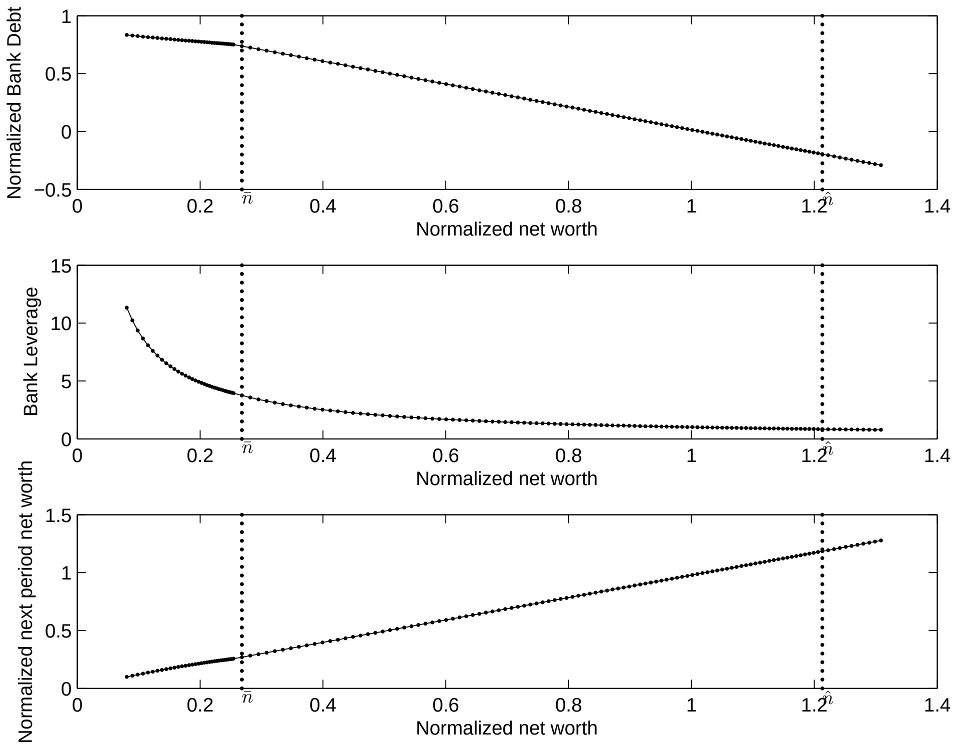

Figure 10 plots the normalized bank debt (top panel), bank leverage (middle panel) and next period net worth (bottom panel) as functions of current period net worth.

Figure 11: **Dynamics of Bank Net Worth**

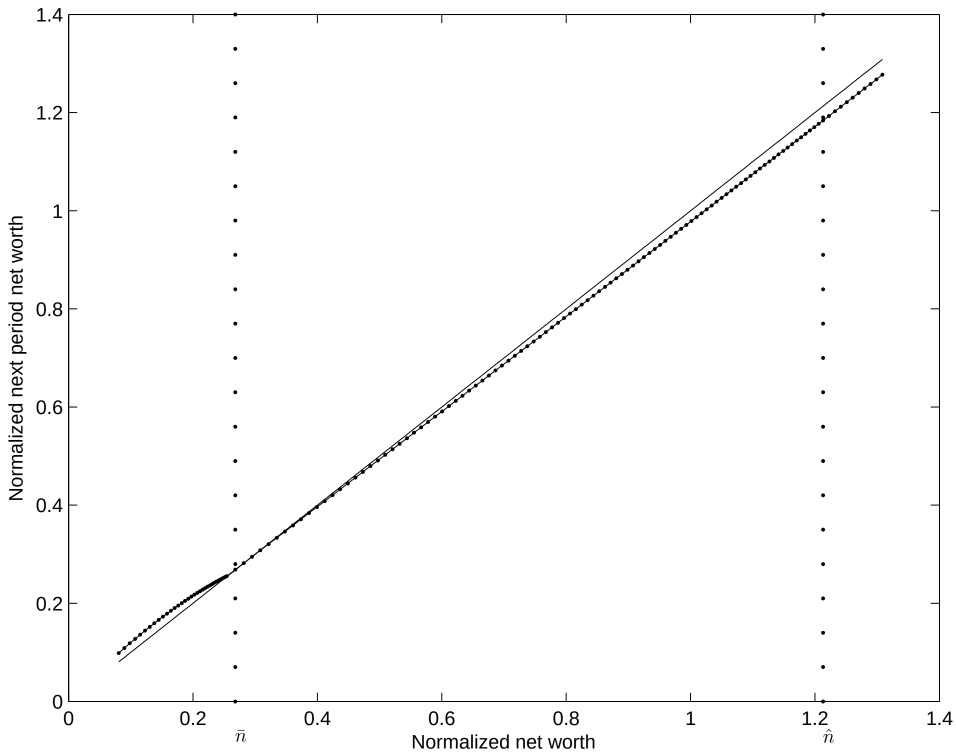

Figure 11 plots next period net worth a function of current bank net worth (dotted line) and the 45 degree line (solid line). The intersection is the steady state level of bank net worth (n_{SSE}).

Figure 12: **Impulse Responses to Productivity Shocks**

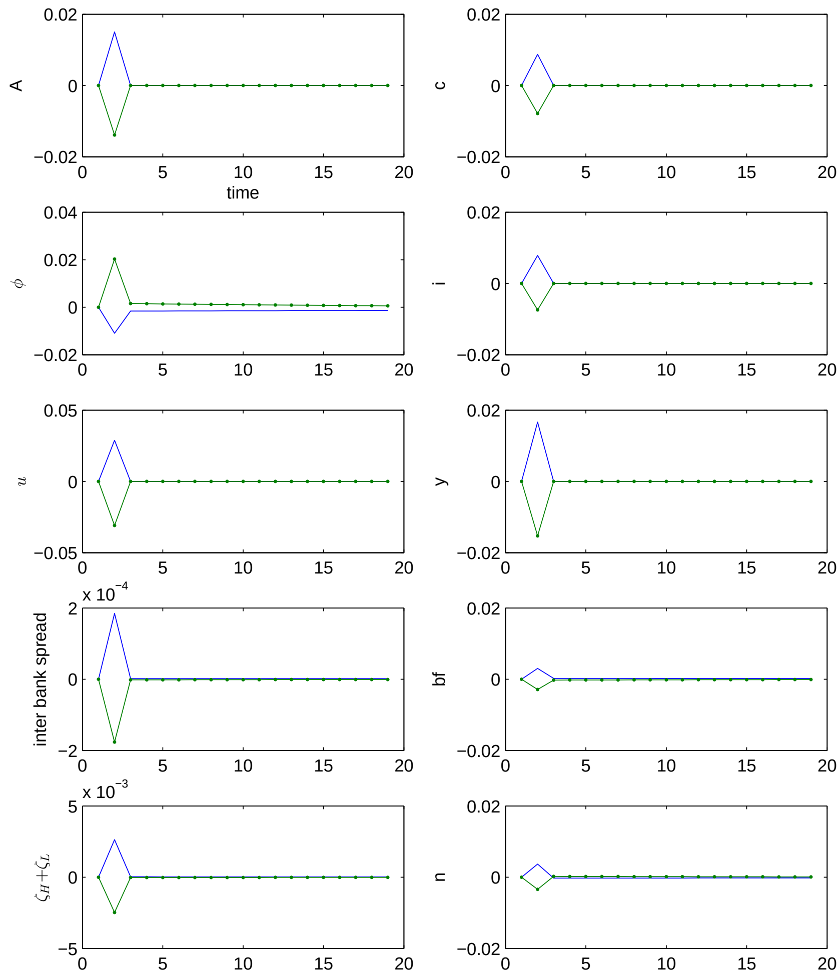

Figure 12 plots the impulse response functions of quantities and prices with respect to a positive innovation in productivity shocks (solid line) and those with respect to a negative innovation in productivity shocks (connected dotted line). We assume shocks are purely transitory and all impulse responses are plotted as deviations from the steady state.

Figure 13: **Impulse Responses to Financial Shocks**

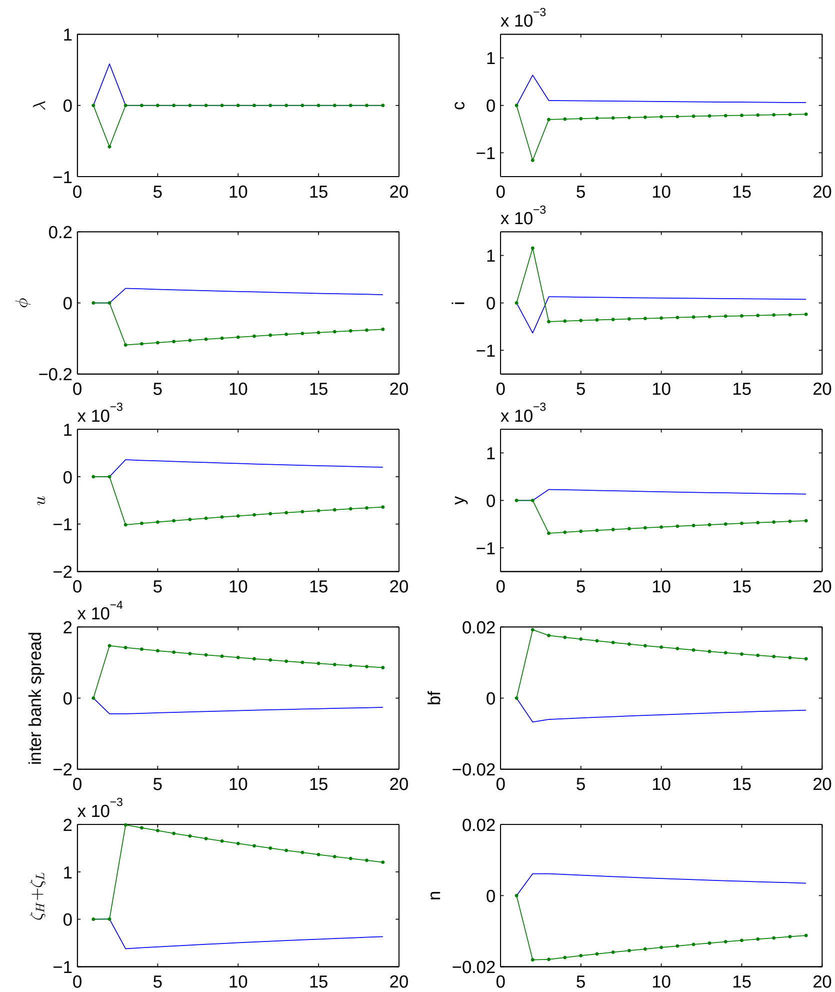

Figure 13 plots the impulse response functions of quantities and prices with respect to a positive innovation in discount rate shocks (solid line) and those with respect to a negative innovation in discount rate shocks (connected dotted line). We assume shocks are purely transitory and all impulse responses are plotted as deviations from the steady state.

Table 1: Calibrated Parameters and Targeted Moments

Parameter		Value	Targeted Moment	Data
β	discount rate	0.994	Investment-output ratio	20%
α	capital share	0.35	Kydland and Prescott (1982)	–
$\frac{\eta}{\eta-1}$	markup	1.25	Hsieh and Klenow (2009)	–
$\frac{a_H}{a_L}$	productivity	2.15	Hsieh and Klenow (2009)	–
δ	depreciation	2.4%	Kydland and Prescott (1982)	–
π	prob. of a_H	0.08	mean capital reallocation	25%
$E[A]$	aggregate productivty	0.1805	mean aggregate growth	1.8%
λ	banker discount rate	0.97	Gertler and Kiyotaki (2010)	–
θ	banker outside option	0.3026	bank leverage	3.67
ω	interbank friction	0.0772	mean TED spread	0.64%
a_0	storage technology	−0.0118	average capital utilization	80%
b_0	storage technology	0.9819	average depreciation	10%
ν	storage technology	0.98	volatility of capital utilization	4.08%

Table 1 lists the parameter values we use in our model and the macroeconomic moments used to calibrate these parameter values.

Table 2: Macroeconomic Moments

Moments	Data	TFP Shocks	Financial Shocks
$E[\Delta \ln Y]$	1.8%	1.85%	1.77%
$Vol[\Delta \ln Y]$	3.49%	3.40%	3.57%
$Vol[\Delta \ln C]$	2.53%	2.45%	1.81%
$Vol[\Delta \ln I]$	13.51%	7.13%	20.42%
$Corr[\Delta \ln C, \Delta \ln I]$	39.7%	92%	28.6%
$AC[\Delta \ln C]$	49%	0.45%	50%
$E[\ln R_f]$	0.86%	4.76%	4%
$Vol[\ln R_f]$	0.97%	0.53%	1.73%
$E[R_I] - E[R_f]$	0.64%	0.74%	0.93%
$Vol[\ln R_I - \ln R_f]$	0.88%	0.04%	0.48%
$Corr[\Delta \ln Y, Var(\Delta \ln Y)]$	-0.15	-0.03	-0.44
$Var[\Delta uf(\phi)] / Var[\Delta TFP]$	$-$	11%	96%

Table 2 documents moments of macroeconomic quantities and interest rates in the U.S. (1930-2009), those generated by our model with TFP shocks, and those generated by our model with financial shocks. $Corr[\Delta \ln Y, Var(\Delta \ln Y)]$ stands for the correlation of output growth and the variance of future output growth. The latter is calculated as the realized variance of quarterly output growth for the next two years. $Var[\Delta uf(\phi)] / Var[\Delta TFP]$ stands for the fraction of variance in output that can be accounted for by changes in the efficiency of capital reallocation.

Table 3: Capital Reallocation and Capital Utilization

Moments	Data	TFP Shocks	Financial Shocks
$E\left[RA/I\right]$	25%	41%	33%
$E\left[Var\left(\ln MPK\right)\right]$	70%	5.13%	5.10%
$Vol\left[Var\left(\ln MPK\right)\right]/E\left[Var\left(\ln MPK\right)\right]$	24%	3.1%	32.3%
$E\left[u\right]$	80%	79.7%	78.3%
$Vol\left[u\right]$	4.08%	1.50%	4.04%
$Corr\left[\Delta\ln\widetilde{TFP}, \Delta\ln RA\right]$	0.24	0.64	0.32
$Corr\left[\Delta\ln\widetilde{TFP}, Var\left(\ln MPK\right)\right]$	-0.14	0.37	-0.48
$Corr\left[\Delta\ln\widetilde{TFP}, \ln u\right]$	0.30	0.96	0.91

Table 3 documents the moments of capital reallocation and capital utilization in the data, and those generated by our models. Our construction of capital reallocation series follows Eisfeldt and Rampini (2006). Details of the calculation of the cross-section dispersion in log marginal product of capital ($\ln MPK$) can be found in Appendix A of the paper. The capacity utilization rate (u) is published by Federal Reserve Bank of St. Louis.

Table 4: Crisis Dynamics

Moments	Non-Recession Periods		Recession Periods	
	Data	Model	Data	Model
$Vol\,[\Delta \ln Y]$	3.53%	3.2%	4.18%	5.96%
$Vol\,[\Delta \ln C]$	2.32%	1.28%	3.42%	2.28%
$Vol\,[\Delta \ln I]$	9.07%	14.08%	9.86%	13.92%
$E\,[u]$	81.1%	81.12%	78.41%	73.23%
$Vol\,[u]$	3.87%	2.87%	5.27%	4.8%
$E\,[\ln R_I - \ln R_f]$	0.56%	0.84%	0.98%	1.62%
$Vol\,[\ln R_I - \ln R_f]$	0.36%	0.17%	0.59%	0.23%

Table 4 documents the first and second moments of macroeconomic quantities and interest rates in recession and non-recession periods in the data and in our model with financial shocks. Both in the model and in the data, recession is classified as two consecutive quarters of decline in real GDP.

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